Detection of ultra-high-frequency variability with a deficit of quanta

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Abstract A technique for detecting harmonics in sparse quantum flows is developed when it is impossible to describe a light curve. A problem that is insoluble in the time representation can be exactly solvable in the Fourier frequency representation. To demonstrate the capabilities of ultra-high-frequency photometry, we present a numerical experiment. We show that at a sampling time of one microsecond on the 2 m telescope, harmonics can be detected in the frequency range up to 500 kHz for an object with U = 14.5. We demonstrate the application of the described technique to the analysis of gamma-ray flare from the Compton Gamma Observatory CGRO. In the BATSE trigger No. 207 in an energy channel of 25-50 keV with flare duration of 0.030 ± 0.002 s, two significant harmonics at 190 and 310 kHz with half-widths of about 25 kHz are fixed, which correspond to velocities of 25,000 km / sec (~ 0.08 speed of light). The size of the object is estimated to be ~ 6000 km, and the size of the active region is ~ 484 km. A possible scenario for gamma ray flare is the merging of a black hole of stellar mass and a neutron star.

Keywords: techniques: photometric, methods: numerical, methods: miscellaneous, gamma-rays: general

1. Introduction

It would seem that the deficit of quanta puts insurmountable obstacles in the way of fast photometry. For the sparse flux of quanta, the concept of the light curve becomes vague. This creates a false impression of an almost insoluble problem when we are dealing with rapidly changing processes. Nevertheless, this unsolvable problem in the time representation can be exactly solvable in the Fourier frequency representation. We can obtain formal mathematical expressions for estimating the frequency, amplitude and phase of a harmonic, starting literally with three photons in a number of measurements. According to Fourier's theorem, there are no restrictions on the absolute values of the mean intensity, sampling frequency, etc. However, the minimum detectable signal amplitude depends on the total number of recorded photons ([1], [7]).

It can theoretically be shown that the minimum amplitude of the harmonic signal a_{min}, detected at the level of confidence probability 3 σ, has the form [7]:

$$a_{min} = < n >^{\frac{1}{2}} \left(\frac{72}{N}\right)^{\frac{1}{4}}$$

where <n> is the average intensity value, N is the number of measurements. Thus, for detecting a harmonic signal, about a hundred real quanta are needed, regardless of the intensity of the source.

To demonstrate the capabilities of ultra-high-frequency (UHF) photometry, we present the following numerical experiment. Let a series of measurements be a Poisson stream of quanta with an average intensity a equal to one ten-thousandth for the sampling time. This means that on average, we need ten thousand measurements to record one quantum, and the remaining data are zeros. The appearance of a quantum is a random process. The series consists of Poisson noise and a sinusoidal wave with an amplitude equal to one ten-thousandth and a period equal, say, half the Nyquist frequency. The length of the series of measurements is N.

$$B = a \cdot \left(1 + \cos\left(2 \cdot \frac{\pi}{4} \cdot (1{:}N)\right)\right) \tag{1}$$

Note that the signal frequency in this case is insignificant, if only it was less than the Nyquist frequency. A usable signal is a non-stationary Poisson process with the intensity described by a sinusoidal wave. The modeling conditions are such that for two hundred thousand measurements there are about two dozen photons. It is quite clear that under these conditions it is impossible to construct the light curve. An example of three such measurements with a time resolution of 1 microsecond is shown in Fig 1 Left.

We will perform formal calculations of the power spectrum of these "observations" to detect a harmonic signal and to estimate its frequency and amplitude. Compare the given signal parameters with the resulting parameters obtained as a result of the simulation.

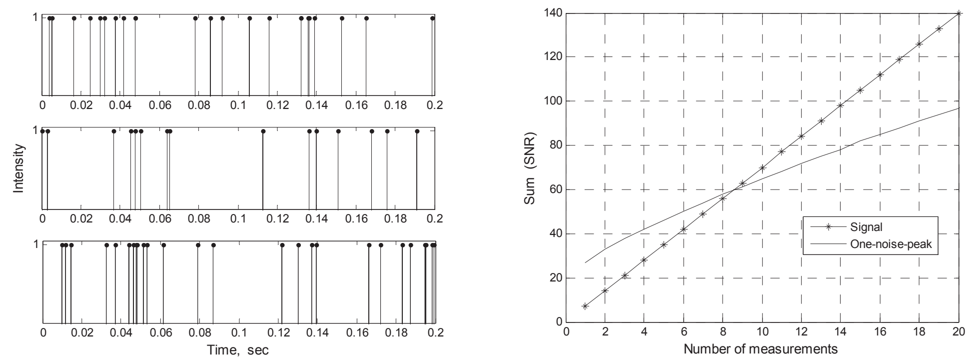

Fig1. *Left: Three arrays of random photocounts selected from the Poisson distribution with the intensity determined by the equation (1) by the amplitude a = 0.0001, the signal sampling time is one microsecond and the length of the series is N = 200000. Right: The cumulative sum of the SNR and snr elements depending on the number of measurements n.*

Define the signal-to-noise ratio (SNR) and detection criterion for the signal:

$$\mathrm{SNR} = \frac{\mathrm{a}^2}{2 \cdot \mathrm{cov}(\mathrm{poissrnd}(\mathrm{B}))} \cdot \mathrm{N} + \mathrm{M}$$

$$\mathrm{snr} = \mathrm{chi2inv}\left(1 - \frac{1}{\mathrm{N}}\right) \tag{2}$$

Here, SNR is the signal-to-noise ratio of a random variable chosen from the Poisson distribution with the parameter *B* (equation 1), *poissrnd* is the random variable generator from the Poisson distribution, *cov* is the variance operator, *N* is the series length, *M* is the average value of the χ^2_2 chi-square distribution with 2 degrees of freedom. The value of *snr* is equal to *chi2inv* - the inverse distribution function, χ^2_2 is a chi-square with 2 degrees of freedom for the quantity (1-1 / N). Its value corresponds to the detection level "One noise peak" in the noise power spectrum.

From a computational point of view, the measurement data can conveniently be represented as an array of power spectra consisting of *n* measurements with sample length *N*. In Fig 1 Right shows the cumulative sum of the *SNR* and *snr* elements, depending on the number of measurements *n*. The *SNR> snr* condition defines the limit value of the number of measurements for the detection of a harmonic signal.

It can be seen from the figure that a sample of about ten such measurements makes it possible to detect a harmonic signal and to estimate its frequency and amplitude.

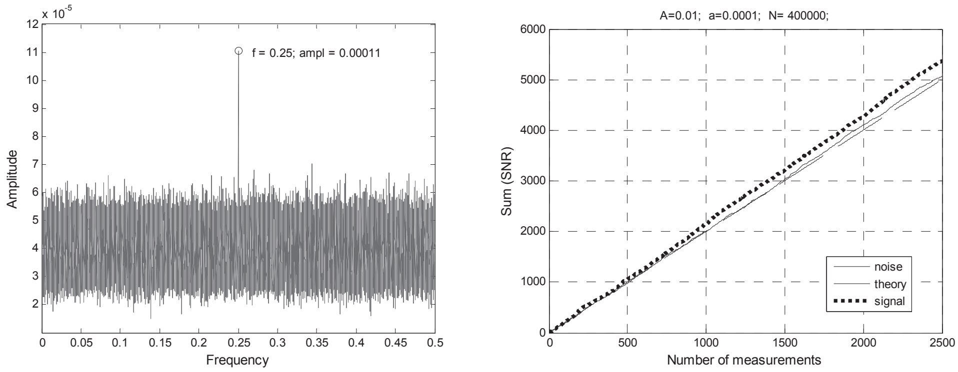

Fig2. *Left: Spectrum of signal amplitudes from equation (1) averaged over 10 samples. Right: The cumulative sum of SNR signals elements and noise peaks (a thin solid line) and its theoretical value is the cumulative sum of chi-square distribution elements with 2 degrees of freedom (discontinuous curve) depending on the number of measurements n.*

Fig 2 Left shows the amplitude spectrum averaged over 10 samples of the signal from equation (1). The specified signal amplitude $a = 10^{-4.}$ The calculated amplitude is $1.1\cdot 10^{-4}$. The dimensionless frequency is equal to the specified frequency f = 0.25. The coincidence of the given and calculated signal parameters proves the effectiveness of the proposed algorithm.

100% variable sources with a Poisson intensity of 10^{-4} counts during signal accumulation can be detected after accumulating only about a hundred photocounts. Conditionally, with a sampling time of one second and the length of *N* = 100000 series, 10 photocounts is expected on average. For a sure detection of harmonics based on the criterion "One noise peak", it is enough to perform about 10 measurements. With a sampling time of one microsecond, the total measurement time is 0.7 seconds. For a 2 m telescope this corresponds to the source U = 14.5. The frequency range is 500 kHz.

Table 1 presents the computation data for sources with Poisson intensity *a* from 10^{-4} to 10^{-5} for a sampling time of one microsecond. The number of quanta during a single measurement is about 80 on the average. At the same time, 100% variable sources can be detected with a brightness of U from 14.5 to 17.0 magnitudes on a 2 m telescope. The length of the series of measurements N ranges from one hundred thousand to one million, the total measurement time is from 0.7 to 9.0 seconds.

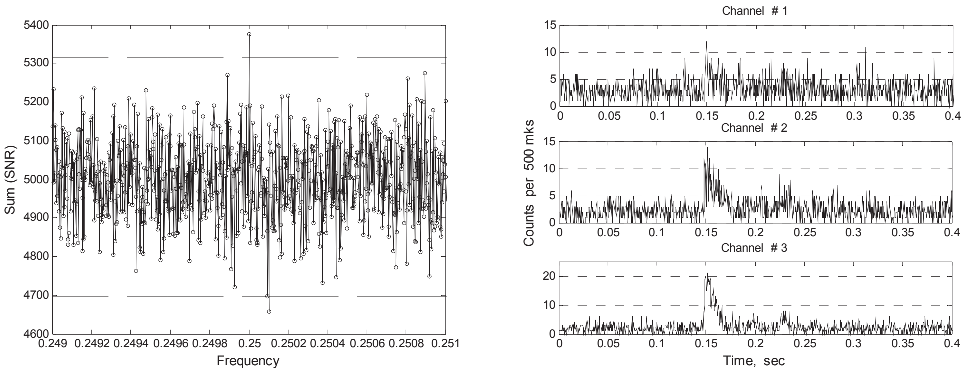

***Fig 3**. Left: The cumulative sum of SNR elements for the critical number of measurements n is shown as solid line. The dashed lines correspond to the detection of a harmonic by the criterion "One noise peak". Right: Light curves with a resolution of 0.5 milliseconds in three energy channels 25-50 keV, 50-100 keV, and 100-300 keV.*

Table 1 Computation data for sources with different Poisson intensity. 100% variable sources.

a	N	n	Nq	U	T, sec
0.0001	100000	7	70	14.5	0.7
0.00009	111111	7	70	14.6	0.8
0.00008	125000	8	80	14.7	1.0
0.00007	142900	8	80	14.9	1.1
0.00006	166666	8	80	15.1	1.3
0.00005	200000	8	80	15.3	1.6
0.00004	250000	8	80	15.5	2.0
0.00003	333333	8	80	14.8	2.7
0.00002	500000	9	90	16.2	4.5
0.00001	1000000	9	90	17.0	9.0

NOTE:
a is Poisson amplitude (average number of quanta during the sampling time of the signal)
N is the length of the sample
n is the number of measurements for signal detection
N_q is the number of quanta in a single sample
U is the magnitude in the U filter
T - total measurement time

2. Stars

Above we considered 100% variable sources. Such sources, apparently, are of mainly methodological interest. Now consider real stars with a 1% variable source. Equation of source:

$$B = a \cdot \left(100 + \cos\left(2 \cdot \frac{\pi}{4} \cdot (1{:}N)\right)\right) \quad (3)$$

We simulate the detection of UHF variability in the range up to 500 kHz with amplitude of 0.01 mag in stars with U equal from 9.5 to 10.8 and for sources in white light W equal to 12.5 to 13.8 mag for the 2 m telescope.

Fig 2 Right shows the cumulative sum of *SNR* elements as a function of the number of measurements *n*. The figure also shows the calculated cumulative sum of elements for noise peaks (a thin solid line) and its theoretical value is the cumulative sum of elements with the chi-square distribution χ^2_2 with 2 degrees of freedom (discontinuous curve). The data correspond to the first line of the elements in Table 2: Poisson amplitude $a = 0.0001$, sample length $N = 400000$, number of measurements $n = 2205$, necessary to detect the harmonic signal, the number of quanta for the time of one measurement $Nq = 88200$, the limiting stellar magnitudes U = 9.5 and W = 12.5, the total measurement time is $T = 882$ seconds.

Fig 3 Left shows the cumulative sum of SNR elements for the critical number of measurements *n*. The dashed lines correspond to the detection of a harmonic by the criterion "One noise peak". Its theoretical value is the cumulative sum of the elements of the distribution χ^2_{2n}, the chi-square with $2{\cdot}n$ degrees of freedom. It can be seen that a signal with a given amplitude $a = 0.0001$ is detected at a dimensionless frequency $f = 0.25$. The coincidence of the given and calculated signal parameters proves the effectiveness of the proposed detection algorithm.

Table 2 Computation data for 1% variable sources with different Poisson intensity.

a	N	n	Nq	U	W	T, sec
0.0001	400000	2205	88200	9.5	12.5	882
0.00009	444444	2231	89240	9.6	12.6	991
0.00008	499998	2256	90240	9.7	12.7	1128
0.00007	571428	2281	91240	9.9	12.9	1303
0.00006	666666	2316	92640	10.1	13.1	1544
0.00005	800000	2350	94000	10.3	13.3	1880
0.00004	999998	2396	95840	10.5	13.5	2396
0.00003	1333333	2456	98240	10.8	13.8	3274

NOTE: Notations as in Table 1

In the next section, we demonstrate the application of the described technique to the analysis of gamma-ray flare from the data of the Compton Gamma Observatory (CGRO).

3. BATSE Trigger No. 207

For our analysis, we used the TTE (time-tagged event) from the BATSE 3B catalog [5], obtained by the Compton Gamma Observatory. Because of the high temporal resolution, the TTE data is suitable for searching for high-frequency variability. Each TTE data set contains the arrival time of all registered photons within 2 μs of time, energy and detector number, in which each photon is registered. The energy boundaries of the channels are approximately 25-50 keV, 50-100 keV, 100-300 keV, and more than 300 keV. We chose one of four short flashes, namely trigger number 207.

The gamma-ray flare of the BATSE trigger No. 207 is fixed in three energy channels No. 1 25-50, No. 2 50-100, No. 3 100-300 keV. The sampling time is one microsecond. For trigger number 207, Cline et al. [2] give flare duration according to the TTE data of 0.030 ± 0.002 s.

Fig 3 Right shows the binned light curves with a resolution of 0.5 milliseconds. In Fig 4 Left initial (raw) light curves in the flare region. The length of the series of measurements is 1,545,916 the number of registered gamma quanta in the three channels is 9447, 7107, 6119. The degree of filling (Poisson's intensity) averages about 0.003. The number of recorded quanta during a flare period is about 300. The light curve "appears" when five hundred samples are combined (0.5 milliseconds, Fig 4 Right).

The duration of the flare in the first energy channel (25-50 keV) is about 20 milliseconds. The smoothed light curve clearly shows the fluctuations in the brightness of the flare with a frequency of about 180 Hz. Variations of brightness at high frequencies can not be traced because of the degradation of the light curve. However, they are clearly manifested in the power spectra (Fig 5 Left). Two significant harmonics are seen at frequencies of 190 and 310 kHz. Harmonics are detected at a spectral resolution of 13 kHz. The spectral resolution is regulated by the choice of the width of the Tukey spectral window when constructing the power spectrum. In the raw unsmoothed spectra, harmonics are not detected. This indicates modulation of the harmonics. Filtering the frequency spectra allows us to set the bandwidth of the modulation.

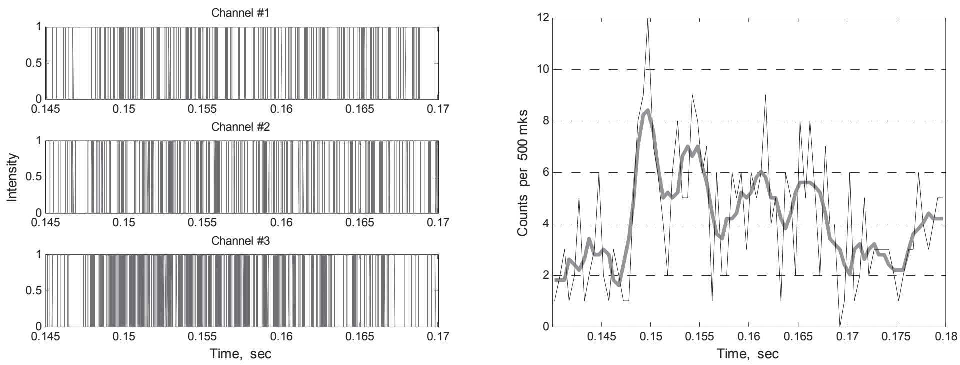

Fig 4. *Left: Initial (raw) light curves in the region of the flare. The degree of filling (Poisson's intensity) averages about 0.003. Right: Smoothed light curve in the first energy channel (25-50 keV). The fluctuations in the brightness of the flare with a frequency of about 180 Hz are clearly visible.*

We constructed the Fourier power spectrum of the first BATSE trigger channel counts of the 207 (25-50 keV) with a time resolution of 1 μs using the technique described above for signal simulation. The spectrum is presented in the form of the signal-to-noise-frequency ratio in the frequency range up to 500 kHz. In the raw spectrum, there are no significant signal peaks visible in Fig 5 Left. We assumed that the signal is subject to strong frequency modulation and used a merger of frequencies to eliminate modulation. This procedure is equivalent to low-frequency filtering in the frequency representation. Since the signal-to-noise ratio (SNR) for noise peaks obeys the χ^2_2 statistic, summing the harmonics within a frequency window of width *n* leads to a χ^2_{2n} statistic for noise peaks.

Fig 5 Right shows the Fourier power spectrum after filtering with a frequency window of width *n* = 10. The detection threshold at the significance level of "One Noise Peak" for the statistics of χ^2_{20} is 49.6. Fig 5 Right shows two significant peaks at 190 and 310 kHz, which coincide with the signal peaks in Fig 5 Left.

Thus, the technique described above in modeling for the detection of harmonic signals in sparse quantum fluxes is confirmed by comparison with the full-scale experiment.
Approximation by the Gaussian allows one to estimate the half-width FWHM of the harmonics peaks at about 25 kHz. The half-widths of the peaks correspond to velocities of 25,000 km / sec (~ 0.08 speed of light). The size of the active region *d* of the oscillation source and the size of the object *D* can be estimated as:

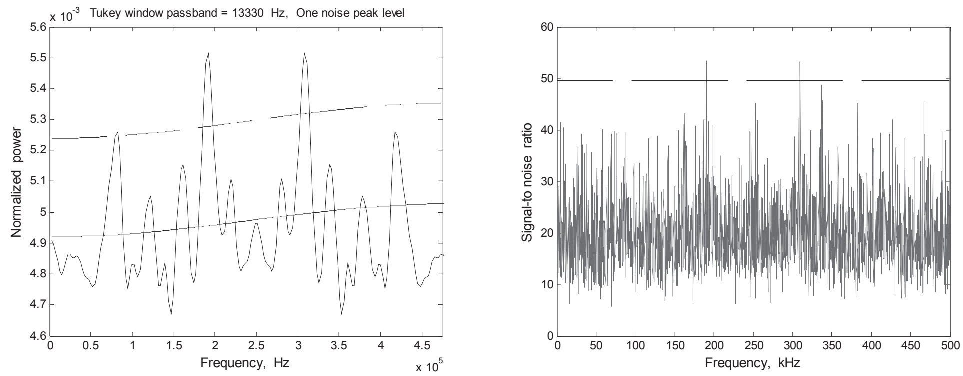

Fig 5*. Left: Power spectrum in the flare region is shown. The dashed curve corresponds to the detection of a harmonic by the criterion "One noise peak". Right: The Fourier power spectrum after filtering with a frequency window of width n = 10. The detection boundary at the significance level "One noise peak" is shown by a dashed line.*

$$d = \Delta f / f \cdot c \cdot \Delta t$$
$$D = c \cdot \Delta t$$

Here *f* and *Δf* are the frequencies of the harmonics and half-widths of the peaks, *c* is the speed of light, and *Δt* is the flare duration (0.02 sec). Thus, the size of the object can be estimated *D* = 6000 km, and the size of the active region *d* = 484 km. These estimates give grounds to consider the object to be relativistic.

A possible scenario for gamma-ray flare is the merger of black holes of stellar mass and neutron stars [8]. During the coalescence process, the substance circulating around the black hole demonstrates rapid fluctuations in the radiation intensity. Such a system will also emit gravitational waves, which lead to a decrease in the radius of the orbit. The time scale of the coalescence process is from several milliseconds to several tens of milliseconds and oscillation frequencies of hundreds of Hz ([4], [3], [6]).

4. Conclusion

We demonstrate a technique for estimating the frequency, amplitude, and phase of a harmonic in sparse fluxes of quanta.

We show that 100% variable sources can be detected after accumulating only about a hundred photocounts. For the 2 m telescope this corresponds to the source U = 14.5. The frequency range is 500 kHz.

For stars with a 1% variable source the detection is achievable for a stellar magnitude U = 9.5 and in white light W = 12.5 for a measurement time of T = 882 seconds.

We demonstrate the described technique to the analysis of gamma-ray flare, BATSE trigger No. 207. Two significant harmonics are seen at frequencies of 190 and 310 kHz.

The half-widths in harmonic spectra are of about 25 kHz, which correspond to velocities of 25,000 km / sec (~ 0.08 of the speed of light). The size of the object is estimated at 6000 km, and the size of the active region is 484 km. These estimates give grounds to consider the object to be relativistic. A possible scenario for gamma flare is the merger of a black hole of the stellar mass and the neutron star

4. References

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