On the possible consequences of multiple phase transitions inside hybrid stars

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Abstract The phase transition from hadron to quark matter can be not a single strong event, but rather a series of weaker phase transitions through intermediate phases (multi-quark states). We perform a phenomenological exploration of this possibility concerning the problem of maximum mass and stability of hybrid stars.

Keywords: Hybrid Stars, Phase Transition, Quark Matter, Stellar Structure

1. Introduction

The density of matter in the central regions of neutron stars can reach more than few times of nuclear density value $\rho_n \approx 2.6\times10^{14}$ *g/ccm*. Hence, a phase transition from hadron matter to quark matter can occur. If this happens, such a star will contain the core, made of quarks, surrounded by ordinary hadron matter envelope. Such stars are called hybrid stars; see review in [1]. These objects have very interesting properties: they can, for example, imitate the properties of ordinary neutron stars. However, they have specific mass-radius diagram peculiarity [2] and they can be a part of the solution for long-standing problem of supernova explosion [3]. Beside this, their properties (if they exist) or their non-existence fact, have a direct connection to maximum neutron (hybrid?) star's mass problem.

With all this topics in mind, we want to refer here to another interesting possibility: in principle, phase transition from hadron to quark matter can proceed in different ways. The ordinary approach considers the single phase transition (Maxwellian or Gibbs type) from hadrons to uniform quark "sea". We want to explore here another "two-steps" opportunity: transition through some intermediate phase, made of multi-quark states [4]. Because of our current lack of knowledge about the properties of such a state, we will work in the frameworks of phenomenological approach which permits to consider the general aspects of the problem without getting too deep into the details.

2. Equation of State

For low-density hadron equation of state (EoS) we use parameterized description from [5] supplied with useful for applications FORTRAN subroutines [6]. This EoS gives for a maximum mass of pure neutron star value well above $2M_\odot$. For a quark EoS we use so-called "constant speed of sound" approximation from [7] (see also discussion in [8]). In this approximation the pressure P of matter is a linear function of its energy-density ε: $P=c_s^2(\varepsilon-\varepsilon_0)$, where ε_0 and c_s are constants, the latter has the meaning of speed of sound in speed of light units. This EoS can easily be connected to the well-known Bag model for quark matter [9]. We use standard value $c_s^2=1/3$ for uniform quark "sea" phase and value $c_s^2=2/3$ for multi-quark phase. The higher value of c_s for intermediate phase is a consequence of general requirement: EoS must become softer with phase transition at growing density. The values of constant ε_0 determine the densities at which phase transitions occur. We assume the simplest Maxwellian type phase transition i.e. constant pressure at coexistence (mixed) phase region.

3. One Phase Transition

Let's start from the standard case of one phase transition (PT). We fix nuclear low-density EoS and sound speed in quark matter to the value $c_s^2=1/3$, and let ε_0 to vary. The change in ε_0 causes the corresponding changes in pressure P_{12} of phase coexistence and, of course, in energy-densities of phases at coexistence ε_1 and ε_2, where index 1 stands for nuclear and 2 for quark matter. It is known that density jump at phase's boundary $\lambda=\rho_2/\rho_1$ is crucial for the stability of a star with a small core of second phase: if λ is greater than critical value $3/2$, the star is dynamically unstable [10]–[11]. This is true for Newtonian gravity. For the case of General Relativity this criterion was generalized by Seidov [12], who showed that here one must use $\lambda=\varepsilon_2/\varepsilon_1$ value for the instability condition and the critical value of λ is now determined as:

$$\lambda_{cr} = \frac{3}{2}\left(1 + \frac{P_{12}}{\varepsilon_1}\right) \tag{1}$$

This expression suggests [13] the use of P_{12}/ε_1 and $\Delta\varepsilon/\varepsilon_1$, where $\Delta\varepsilon=\varepsilon_2-\varepsilon_1$, as coordinates for global exploration of mass-radius curves of hybrid stars with different EoSes. Here x-coordinate, $P_{12}/\varepsilon_1 = P_1(\varepsilon_1)/\varepsilon_1$ is directly connected with the density of the phase transition beginning, while y-coordinate, $\Delta\varepsilon/\varepsilon_1=\lambda-1$, characterizes energy-density jump. The example of calculation is shown on the ***Fig1.***

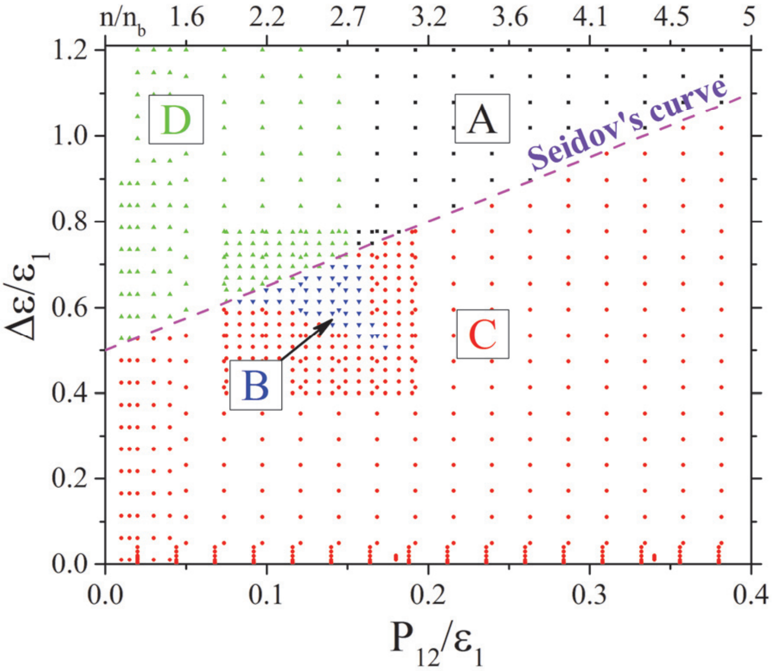

Fig1. *Topology diagram for mass-radius curves of hybrid stars. Upper axis shows the value of baryon density at the beginning of phase transition in units of nuclear baryon density* $n_b \approx 0.16\ fm^{-3}$.

The most interesting fact about this graph is that all its space is subdivided onto four zones with different hybrid star's mass-radius relation topology (see more details in [13] and [8]). The A (Absent) type is in the upper-right part of the diagram (colored black), above the Seidov's curve (the line of critical λ according to Eq. (1)). In this domain of parameters stable hybrid stars are absent. Below Seidov's curve there is a region of topology type C (Connected, colored red): stable are the hybrid stars with $0 \le M_q \le M^{max}$, where M_q is the mass of quark core of a star, and M^{max}– some maximum value, which depends on the point of the diagram. Upper-left region – type D (Disconnected, green) is characterized by the stability of hybrid stars with mass of quark core $0 < M^{min} \le M_q \le M^{max}$ i.e. there are maximum and minimum values for M_q. And last type B (Both, blue) – is a mixture of types C and D, i.e. on the mass-radius diagram there exist the stable hybrid stars with small quark cores and with big quark cores, but there is an unstable gap between. This diagram is very useful for the exploration of the ranges of EoS parameters and hybrid star properties. ***Fig2*** below shows two examples.

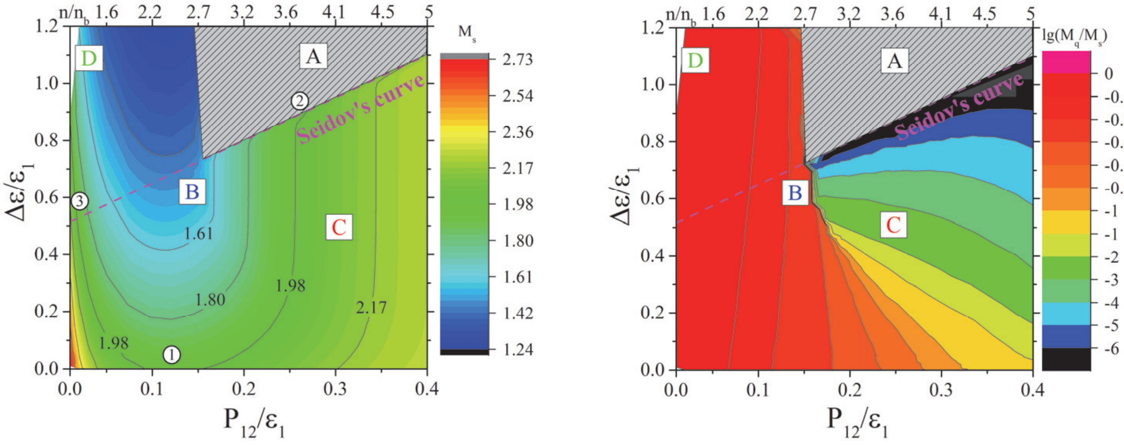

Fig2. *Left: level lines of maximum mass of hybrid star* M_s*. Right: level lines of* $lg(M_q/M_s)$*, where* M_q *is the mass of quark core in maximum mass stellar configuration. Area of type topology A left empty.*

On the left panel we plot level lines of maximum mass of hybrid star M_s. Area for A type topology left empty. Lines of constant mass have a quasi-parabolic structure. Now the most precisely measured value of neutron star mass is $1.97 \pm 0.04 M_\odot$ [14]. In view of this restriction we see a few allowable domains on the diagram. First, this is almost a full range of M_s at low x-coordinate (i.e. low densities of PT beginning). In addition, in lower left corner of the diagram there is a domain with very high masses of hybrid stars (see also [8]). Next, the region with $\Delta\varepsilon/\varepsilon_1 \approx 0$ is also favorable for high maximum mass. This is a natural result, because low $\Delta\varepsilon/\varepsilon_1$ means a weak phase transition. Second, there is a high P_{12}/ε_1 area of type C topology. In addition, three points marked by numbers 1, 2, 3 inside of small circles are shown and discussed below in subsection 4.2.

Now let's take a look at the right panel of ***Fig2***. Here we plot level lines of $lg(M_q/M_s)$, where M_q is the mass of quark core in maximum mass stellar configuration. One can see that unfortunately all the high-x region of type C topology has very low relative quark core mass. This means that the stability of high-mass hybrid stars here is illusive in some sense: only tiny part of the star can persist in quark matter state and the star is almost pure neutron one.

4. Two Phase Transitions

4.1. The splitting concept

Our work on multiple phase transitions was first motivated by the conclusion steamed from variational principle for the stars with PT [15]. In the cited paper we showed that the weak “splitting” of one phase transition into two parts is always favorable for stability of the star, assuming the total energy-density jump $\lambda_{tot}=\lambda_1\lambda_2$ remains the same. Now we present here the results of our numerical calculations of PT splitting for various conditions and splitting strength.

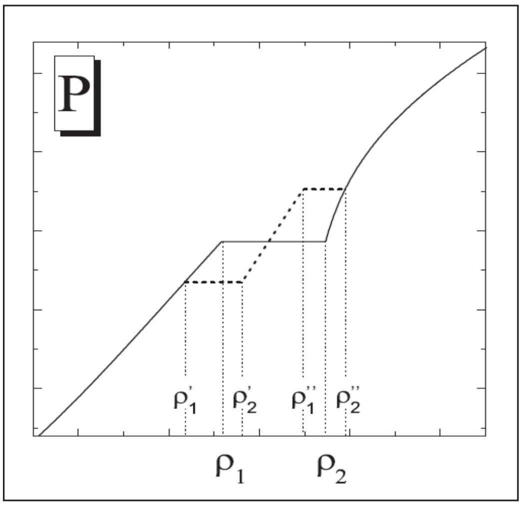

Fig3. *Pressure-density dependence for original (solid line) and split (dashed) EoS.*

First, we fix the parameters of base EoS, i.e. the properties of uniform “quark sea” phase (the parameters of low-density nuclear EoS are the same for all cases). Thus we choose the concrete point on our base topology diagram (***Fig1***). Next we start to split this EoS by inserting intermediate quark phase with $c_s^2=2/3$ between original phases. This process is illustrated by ***Fig 3***, where solid line shows pressure-density dependence for one-PT case, while dashed line shows split EoS. Outside the splitting region this two EoSes coincide. The parameters of intermediate EoS is convenient to fix with the aid of two multipliers μ_1 and μ_2:

$$\rho_1^{\prime} = \mu_1\rho_1,\ \mu_1 \le 1,$$
(2)

$$\rho_2^{\prime\prime} = \mu_2\rho_2,\ \mu_2 \ge 1. \tag{3}$$

With multipliers μ_1 and μ_2 in hand, other parameters, such as $\rho_2^{\prime}$ and $\rho_1^{\prime\prime}$ (see ***Fig3***) are calculated automatically from the phase coexistence equations. This is also true for intrinsic EoS parameters of the inserted phase.

4.2. Examples of calculation

Now we present a few examples of calculations for different initial conditions. First, we take a point with coordinates *(0.12,0.05)* on the topology diagram (topology type C). This point is shown by rounded number “1” on the left panel of ***Fig2***. It has the value of maximum mass $1.91M_{\odot}$ for stable hybrid star with one PT. The original phase transition is weak here, but still enough to lower the maximum mass value below $2M_{\odot}$ mass limit.

The results of our splitting procedure are shown on the combined ***Fig4***. The left panel of it shows multipliers $(\mu_1$-$\mu_2)$ diagram with color map for maximum mass of hybrid star with corresponding splitting (see color bar on the right). One can see that yellow, orange and red domains of $(\mu_1$-$\mu_2)$ diagram satisfy the observational restriction for maximum mass [14]. On the right panel of ***Fig4*** we plot Mass-Radius curves for selected cases (the values of split parameters are shown on the curves). Central density of a star increases when moving along a curve from right to left (and, correspondingly, from bottom to top). Rightmost line corresponds to one-PT case. Black line corresponds to nuclear (hadron) phase inside the center of a star, red – to original "quark sea" phase, and blue color – to the inserted quark phase. It's clear that, from one hand, splitting can increase the maximum mass well above its critical value. But from the other hand, right panel of ***Fig4*** shows that the hybrid stars with mass above limit consist mostly of inserted quark phase, with small original quark phase core and tiny hadron envelope. This result is in some sense negative. To end with this case, we mention the specific peculiarity of *M-R* diagram that clearly can be seen on the right panel of ***Fig4***: all the *M-R* curves pass through the same small region at $R \approx 11$ km and $M \approx 1.9M_{\odot}$. We explained this remarkable fact in [2].

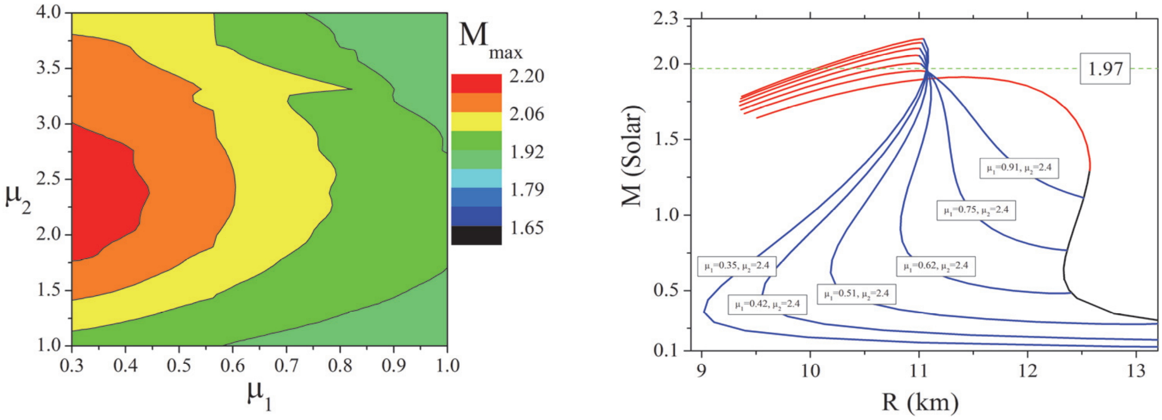

Fig4. *Left: the multipliers (μ_1-μ_2) diagram for (0.12, 0.05) point. The color map shows maximum mass of hybrid star with corresponding splitting. Right: Mass-Radius curves for selected cases, the values of split parameters are shown on the curves. Green horizontal line shows observational limit [14]. Rightmost curve corresponds to one-PT case.*

Now we can move to two other interesting cases, marked by rounded numbers "2" (topology type **A**) and "3" (type **D**) on the left panel of ***Fig2***. The first one corresponds to "Absent" topology, i.e. no stable hybrid configuration for one-PT case. ***Fig5*** (left panel) shows the (μ_1-μ_2) diagram for this case. We see that not only stable hybrid stars appear, but there exist (μ_1,μ_2) combinations which even fulfill $1.97M_{\Theta}$ observational constraint. The empty domains of diagram correspond to forbidden multiplier combinations, for which no PT can be found. But again only tiny cores of original phase are exists inside the stable configurations.

Third example, point "3", corresponds to low density of one-PT beginning and energy-density jump exceeded the Seidov's limit. Maximum stable mass here is $1.9M_{\Theta}$. The (μ_1-μ_2) diagram for this case is on the right side of ***Fig5***. We see here a different topology of M_{max} level lines, compared to the previous cases. And again the area of consistent with observations masses and correspondent multipliers is rather wide. But now the core of original phase here can be not so small: its maximum relative value $M_q/M_s \approx 0.3$ for the stable hybrid configuration with maximum mass $1.97M_{\Theta}$. But the hadron envelope is very small here anyway.

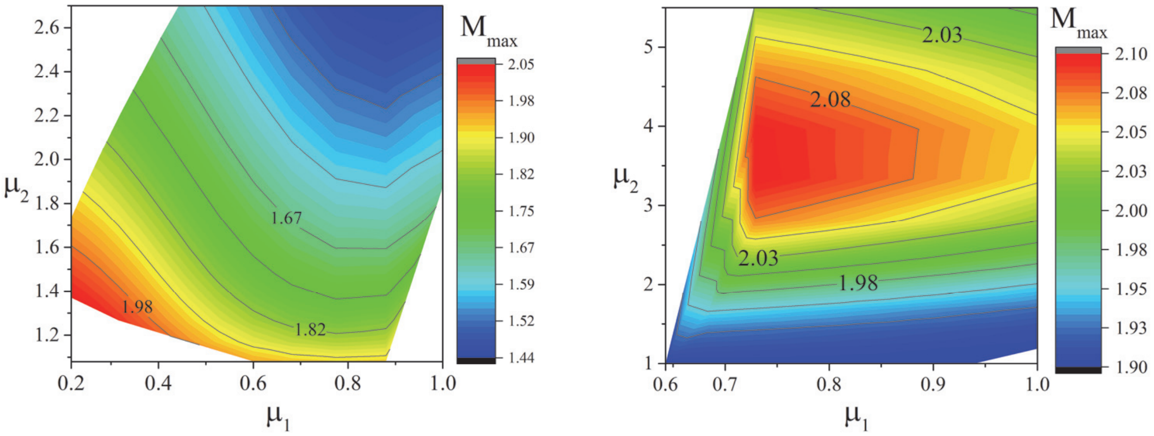

Fig5. *Multipliers (μ_1-μ_2) diagram for (0.26, 0.9) point (left) and (0.02, 0.58) point (right). The position of these points on the topology diagram is shown on the left panel of* ***Fig2*** *as points "2" and "3" correspondingly.*

5. Conclusion

In this work we explore the idea of multiple phase transitions inside hybrid stars. Our main purpose was to answer the question: can the insertion of intermediate quark states stabilize the star against collapse to black hole and thereby to increase the maximum stellar mass. Because of our current lack of knowledge about the properties of matter at very high densities, we choose the phenomenological approach (and more specifically, the constant speed of sound approximation for EoS of quark matter) as a method of investigation. The results of our research are twofold. The good news is that we really can reach and even overcome the observational $1.97M_{\odot}$ limit. Even the domain of original EoS parameters without any stable hybrid branch (type **A** topology) can be converted by specific splitting to stable hybrid configuration. But the bad side of this is that our new multi-phase configurations with observationally acceptable properties are, as a rule, almost pure quark stars with only a tiny envelope, made of hadron matter. Now is hard to decide if this is a generic property of any possible phase transition scenario or the result of our simplified approach. We plan to investigate this in close future with the aid of additional EoS parameters variation, what can be done by easy generalization of constant speed of sound approximation.

Acknowledgements

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