Localization of gravitational waves as a test of gravitation theory

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Abstract Detection of the gravitational wave events by Advanced LIGO antennas has opened the new possibility for the study high energy astrophysical processes and also fundamental physics of the gravitational interaction. A new method is presented for measuring the polarization state of an incoming GW by using localization of GW sources along the apparent circle of a detected event. The method takes into account the antenna-pattern functions for different polarization modes and the the detected strain ratio. In is shown that the apparent circles on the sky for allowed positions of the GW sources for the GW150914, GW151226 and LVT151012 events are parallel to the plane of the disc-like large scale structure known as the Local Super-Cluster (LSC) of galaxies which extends up to radius ~100 Mpc and having thickness ~30 Mpc.

Keywords: Gravitational Waves, Detection of gravitational waves, Source localization

1. Introduction

In the 1980's, the Laser Interferometer gravitational wave Observatory (LIGO) was proposed for detecting gravitational waves with the principal goal to study astrophysical GWs and stimulate research in fundamental physics concerning the nature of gravity. Recent detection of gravitational wave signals by Advanced LIGO antennas has opened such possibility for study physics of the gravitational interaction.

In the situation when there is no reliable optical (and other electromagnetic bands) identification of the GW events, the interpretation of the physics of the GW source is still uncertain. Even though the model of tens solar masses binary black holes coalescence at the distance 400 - 1000 Mpc is consistent with existing GW data (Abbott et al. [2016a], Abbott et al. [2016b]), one should also test alternative possibilities which are allowed by modern theories of the gravitational interaction (Eardley et al. [1973], Maggiore and Nicolis [2000], Will [2014], Baryshev [2017]).

Here we develop a method based on very general physical arguments, which allows us to distinguish between tensor and scalar polarization states predicted by the scalar-tensor gravitation theories. In particular, for the case of two antennas actual position of the GW source at the apparent circle on the sky can be used for determination of the polarization state of the detected GW. Hence, our method allows testing astrophysical models proposed for explanation of the physical processes generating the gravitational waves.

2. Polarization states of a gravitational wave in the modern theories of gravity

There are two main approaches to study the physics of gravitational interaction: the geometrical one, known as General Relativity Theory (hereafter GR) proposed by A. Einstein, and the field theory approach introduced by R. Feynman. In the frame of both approaches, there is predicted the existence of gravitational waves (hereafter, GW) as a result of massive bodies interaction.

The first approach is the geometrical Einstein's general relativity theory (GRT, also called "geometrodynamics"), which is based on the concept of metric tensor g^{ik} of curved Riemannian space-time (Einstein [1916], the standard textbooks on GRT: Landau and Lifshitz [1988], Misner et al. [1973]). The second approach, the Feynman's field gravitation theory (FGT, also called "gravidynamics") is a non-metric relativistic quantum theory, where the gravitational interaction is described by a symmetric second rank tensor potential ψ^{ik} in the Minkowski space-time (Feynman [1971], Feynman et al. [1995]).

A gravitational wave is characterized by its polarization state, which reflects the response of test bodies on the propagation of the wave. Modern theories of gravitation predict the existence of several GWs modes generally divided on tensor and scalar, longitudinal and transverse. A certain polarization state of a received GW significantly depends on the nature of a GWs source. In particular, a merging binary system of stars is expected to give a GW with a tensor transverse polarization state but not a scalar one, whereas a symmetric core-collapse supernovae might be capable to produce a scalar GW mode without a tensor component.

On the other hand, a polarization state is limited by circumstances of a chosen theory. Thus, GR takes under consideration only tensor transverse modes, whereas some modified GR theories as well as field gravitation theory apart from tensor consider also the existence of scalar modes. Therefore, the detection of a certain polarization state of an incoming GW provides a test on a theory of gravitation.

3. Interferometric antennas and polarization states of GWs

Let us consider a GW antenna based on a Michelson-type interferometer with two orthogonal arms having four test masses at their ends. The receiver is at the rest in the local proper reference frame, with the origin of spatial coordinates in the corner of the system and the *X* and *Y* axes along the antenna's two arms, ***Fig1***. The GW passing through the antenna displaces the test masses, thereby changing the length of each arm from its initial length L_0 (for LIGO detectors L_0 = 4 km). The monitored by laser difference between lengths of these arms $\Delta L(t) = L_X - L_Y$ normalized by the initial length of the arm L_0 gives the observed at the antenna strain $\bar{h}(t) = \Delta L(t)/L_0$.

For laser interferometers, the general form of the signal $\bar{h}$ is a composition of antenna-pattern functions F with waveform *h(t)* for the corresponding polarization mode (Will [2014]):

$$\bar{h}(t; \zeta,\Phi,\Psi) = F_{SL}(\zeta,\Phi)h_S(t) + F_{ST}(\zeta,\Phi)h_S(t) + F_+(\zeta,\Phi,\Psi)h_+(t) + F_\times(\zeta,\Phi,\Psi)h_\times(t) \quad (1)$$

Waveforms *h(t)* are critically dependent on the nature of a source and the processes producing this GW, whereas antenna-response functions $F(\zeta,\Phi,\Psi)$ depend only on the position of the source on the sky relative to the antenna characterized by zenith angle ζ and azimuth Φ in the horizontal coordinate system of the antenna. Therefore, for the same source, a received by several antennas GW with a certain waveform has different antenna response functions.

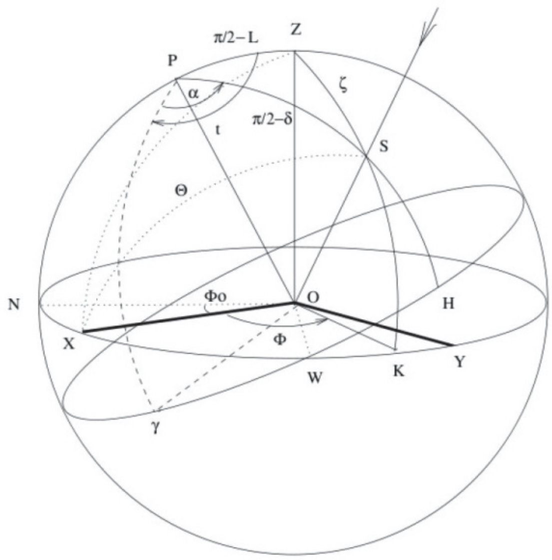


Fig1. *Equatorial and horizontal coordinate systems of an interferometric antenna for the GW source S. Z is the zenith, P – the northern pole, γ defines the sidereal time, α – the right ascension (RA), δ – the declination (DEC), ζ – the zenith angle. The reference direction of the detector is the direction OX with the azimuth Φ0.*

3.1. Antenna-response functions of two-arms interferometric antennas

Antenna-pattern functions $F(\zeta,\Phi,\Psi)$ are determined for each polarization state (see, e.g., Will [2014]) by the angles (ζ,Φ) of a source position in the horizontal coordinate system (hereafter CS) of an antenna, ***Fig1***.), and polarization angle Ψ in the case of tensor GW (for definition see p. 366 Hawking and Israel [1989]):

$$F_{+}(\zeta,\Phi,\Psi) = \tfrac{1}{2}\,(1+\cos^2\zeta)\cos 2\Phi \cos 2\Psi - \cos\zeta \sin 2\Phi \sin 2\Psi \quad (2a)$$

$$F_{\times}(\zeta,\Phi,\Psi) = \tfrac{1}{2}\,(1+\cos^2\zeta)\cos 2\Phi \sin 2\Psi - \cos\zeta \sin 2\Phi\, \mathrm{cow} 2\Psi \quad (2a)$$

It is convenient to represent these functions in three dimensions as beam patterns of a detector, Figs 2a, 2b. From these pictures, it can be seen clearly that beam patterns for tensor cross × and plus + polarization states are different, therefore, an interferometric (two-arms) antenna-detector allows us to distinguish between these polarizations.

Regarding antenna response on a scalar polarization state, which might be longitudinal (SL) or transverse (ST), it can be shown that the functions are different only by a sign Will [2014]:

$$F_{SL}(\zeta,\Phi) = \tfrac{1}{2} \sin^2\zeta \cos 2\Phi \quad (3a)$$

$$F_{ST}(\zeta,\Phi) = -\tfrac{1}{2} \sin^2\zeta \cos 2\Phi \quad (3b)$$

Therefore, beam patterns of an interferometric antenna are the same for scalar longitudinal and transverse polarizations, Fig.2c It means that it is not possible to recognize scalar modes by means of a typical two-arms interferometer.

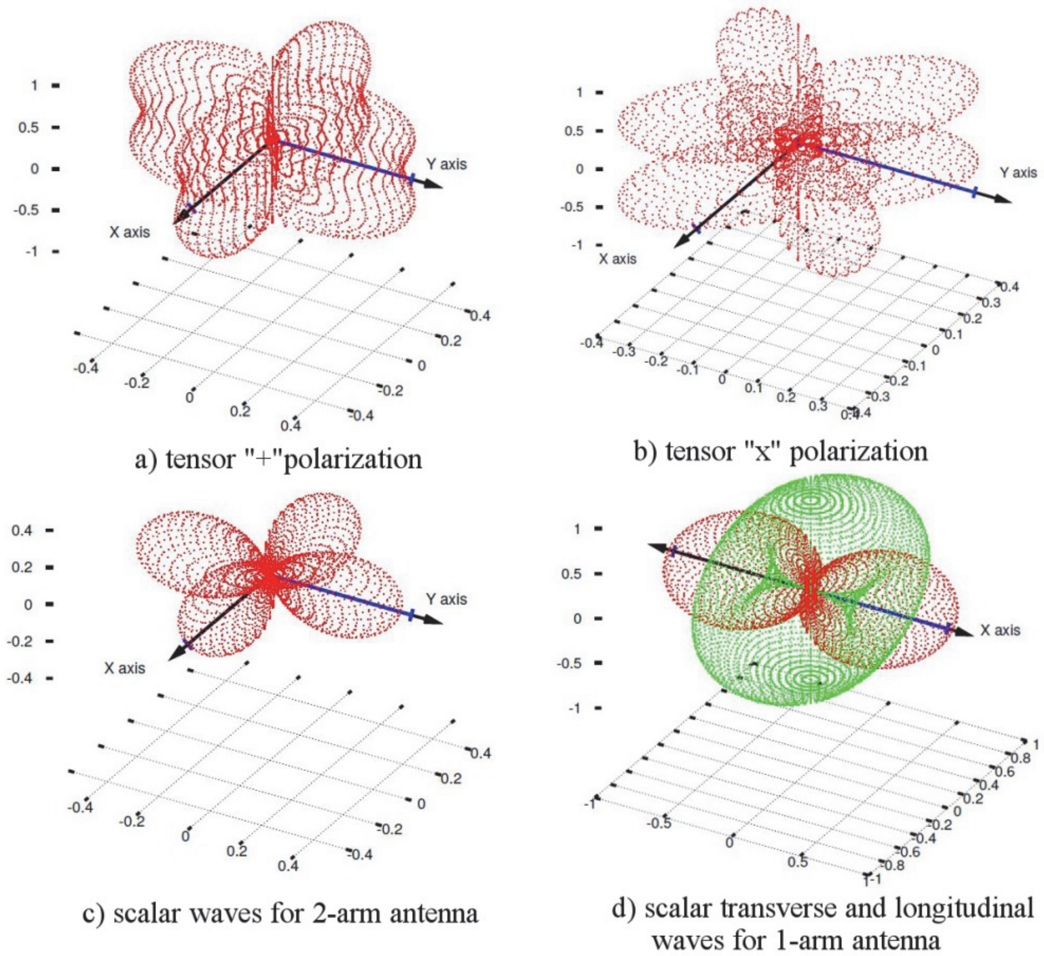


Fig2. *Antenna beam patterns for different polarizations of an incoming GW. Blue lines indicate the arms of the detectors along the X and Y axis. Red points are the beam pattern depending on the location of a GW source on the sky. Green points in (2d) are the antenna response for scalar transverse wave in the case of one-arm mode.*

3.2. Antenna-response functions of one-arm interferometric antennas

To tackle the problem of indiscernibility between scalar longitudinal and transverse modes by a two-arms interferometer, there has been proposed a modification of an interferometric detector as one-arm antenna (one-arm mode) having one working arm with two test masses Baryshev and Paturel [2001]. Then, the observed strain is given by the length change of the working arm (X-axis) relative to the length L_0 of the fixed (former Y-axis) arm: $\Delta L(t) = L_X - L_0$. The amplitude of the arm-length variation $h^0 = \Delta L_{max}/L_0$ can be used as a normalization constant.

The antenna-response functions for scalar longitudinal and transverse modes are the following (Baryshev and Paturel [2001]):

$$F_{SL}(\zeta,\Phi) = \cos\Theta = \sin\zeta\cos\Phi \qquad (4a)$$

$$F_{ST}(\zeta,\Phi) = \sin\Theta \qquad (4b)$$

where Θ is the angle between the direction of a GW propagation and X-axis of the antenna, Fig1.

The beam-patterns are depicted in Fig. (2d), where the red points indicate the response of the antenna on the scalar longitudinal and the green – on the scalar transverse. Thereby, applying one-arm modification of the interferometric antenna, it is possible to recognize scalar longitudinal and transverse polarization modes.

4. Method description

The proposed method for source localization is based on the considered properties of antenna-response functions and the possibility to recognize a polarization state of an incoming GW. The method uses the value of the maximal amplitude of the detected strain *h(t)* at each antenna in the network during so-called "pre-coalescence" phase, sidereal time (ST) of the signal arrival, time delay Δ between signal registration at the antennas as well as the position of antennas in the equatorial CS.

The detected time delay Δ between registrations determines a radius of an apparent circle (hereafter AC) on the unit sky sphere, along which the source of GW might be located. The centre of the AC is defined by the direction of the vector joining the two antennas at the sidereal time (ST) of the event. In the equatorial CS, each point at the AC is defined by right ascension (RA) α and declination (DEC) δ. Regarding the detector, the considered point as possible source S has horizontal coordinates: zenith angle ζ and azimuth Φ, Fig1. The antenna-pattern functions $F(\zeta,\Phi,\Psi)$ are different for distinct antennas in the network.

***Table 1**. Detection parameters of LIGO events. ST is the sidereal time of the event, h0 – the strain as the maximal amplitude normalized by 10-21, ΔLH – the time delay between registrations at Livingston and Hanford antennas. ST is the sidereal time of the event given in hours.*

GW event	(UTC)	ST	Δ_{LH} [ms]	h^0
GW150914	(09:50:45)	3.33	$6.9^{+0.5}_{-0.4}$	0.6
LVT151012	(09:54:43)	5.24	– 0.6±0.6	0.3
GW151226	(03:38:53)	3.89	1.1±0.3	0.3
GW170104	(10:11:59)	11.1	$-3.0^{+0.4}_{-0.5}$	0.3

A typical detected strain *h(t)* can be theoretically decomposed into the purely time-dependent part, which is a waveform *s(t)*, and time-independent part or geometrical factor $G(\zeta,\Phi,\Psi)$, which depends only on the position of the source in the horizontal CS of an antenna.

$$h(t) = \frac{\Delta L(t)}{L_0} = h^0 s(t) G(\zeta,\Phi,\Psi) \quad (5)$$

where h^0 is the amplitude of the signal.

The geometric factor $G(\zeta,\Phi,\Psi)$ is determined by the relative orientation of an antenna with respect to the position of the source on the sky at the sidereal time (ST) of the detection, angles (ζ,Φ), as well as by the polarization angle Ψ for tensor GW. In the particular case of an incoming GW in a single polarization mode, the G-factor is equivalent to the antenna-pattern function ($G \equiv F$) for this mode. In a general case, the G-factor represents a composition of antenna-pattern functions F weighted by coefficients identifying the entering polarization states (1).

Regarding the time-dependent part of a strain, it is worth to mention that the normalized waveform *s(t)* depends only on the nature of the source and, consequently, is the same at each antenna in the network for a particular GW event.

Then the following relation holds at the fixed sidereal time:

$$\frac{h_1}{h_2} = \frac{G_1}{G_2} = \frac{G(\zeta_1,\Phi_1,\Psi_1)}{G(\zeta_2,\Phi_2,\Psi_2)} \tag{6}$$

where "1" and "2" indicate the values related to the considered antennas.

Thus, the calculated ratio of G-factors G_1/G_2 for a certain point on an AC predicts the ratio of the strains h_1/h_2, which should be observed in the case of an incoming GW of a certain polarization state. In other words, it means that a point at the AC being the real source of a GW of a certain polarization should produce the ratio of geometrical factors, which is equal to the ratio of the strains h_1/h_2. Therefore, it is possible to highlight such points on the AC, where the calculated G_1/G_2 is approximately equal to the observed h h_1/h_2. This is the main principle of the method.

It is important to emphasize that the detected strain ratio does not depend on the nature of the source but only on the antenna position relative to the source and polarization state of the incoming GW.

5. Application of the method for the source localization of events

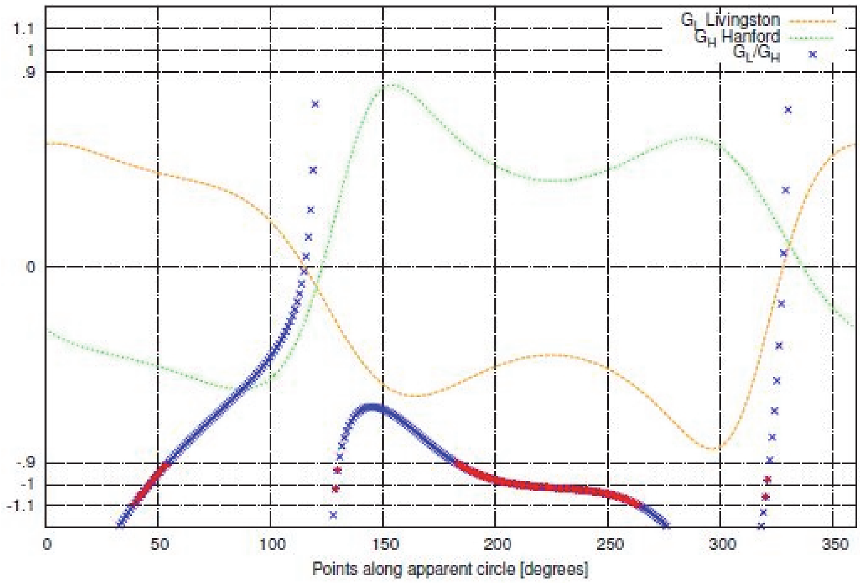
In order to show the method application, let us consider three GW events 2015: GW150914, GW151226 and LVT151012 (Abbott et al. [2016a]), as well as the event GW170104. The used data are given in Table1. For the reported GW events, there were only two antennas in operation: LIGO Hanford and Livingston. The time delay ΔLH between the signal registration at these antennas together with the sidereal time ST of the event determine the AC on the sky, along which the GW sources will be searched.

To select the points of possible GW localization for each event, there should be calculated the G-factors and their ratio for the assumed polarization states of the GWs and compared with the observed ratio of the strains at the antennas couple. The points, for which the ratio G_L/G_H for the considered polarization states is approximately equal to the observed h_L/h_H with the error of observations ≈ 1.0 ± 10%, are highlited by red in Fig. 3 for a tensor transverse plus polarization and for scalar polarization.

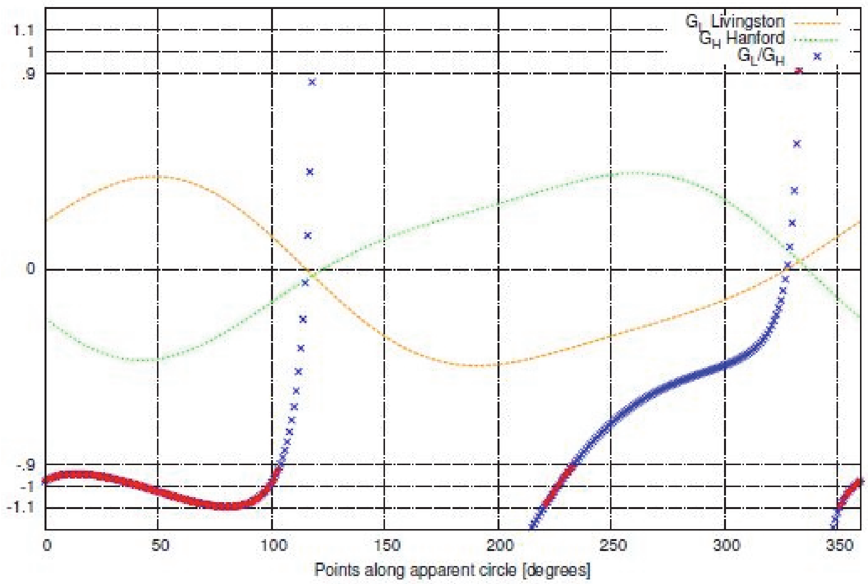
5.1. Representation of the results in the supergalactic CS

Besides the most common representation of the GW source localization in the equatorial CS, there can be taken into account other coordinate systems, which are capable to show the possible coincidence of the source localization with the galaxies distribution.

In particular, there has been introduced the supergalactic (hereafter SG) coordinate system (see eg., Courtois et al. [2013]) for the convenience of considering the Local Super-Cluster of galaxies (hereafter LSC). The LSC has the _lamentary disc-like structure with the radius ~ 100 Mpc, the thickness ~ 30 Mpc and the centre roughly in the Virgo cluster (*SGL* = 104º; *SGB* = 22º), and the North Pole *SGB* = 90º with galactic coordinates *l* = 47.37º, *b* = 6.32º (Courtois et al. [2013], de Vaucouleurs [1953], de Vaucouleurs [1958], di Nella and Paturel [1995]).



for tensor ”+” GW



for scalar (transverse or longitudinal) GW

Fig3. *G-factors for possible polarization states of GW150914. The orange and green thin curves represent the G-factors along the AC calculated with LIGO Livingston and Hanford coordinates respectively. Blue dots curve shows the ratio GL/GH. The allowable places of the GW source are highlighted by red corresponding to the condition GL/GH ≈ 1 ± 10%.*

In Figs. (4, 5) the LSC is represented in SG CS with the background projection of the 2MRS catalogue of galaxies, which is the result of the 2MASS all-sky IR survey (Huchra et al. [2012]) and includes the redshifts of 43 533 galaxies. There have been taken the subsample containing 32 656 galaxies with $z \leq 0.025$, which corresponds to the spatial distribution of the galaxies within ~ 100 Mpc known as the Local Super-Cluster or Laniakea Super-Cluster or Home Super-Cluster (hereafter LSC, de Vaucouleurs [1958], di Nella and Paturel [1995], Tully et al.).

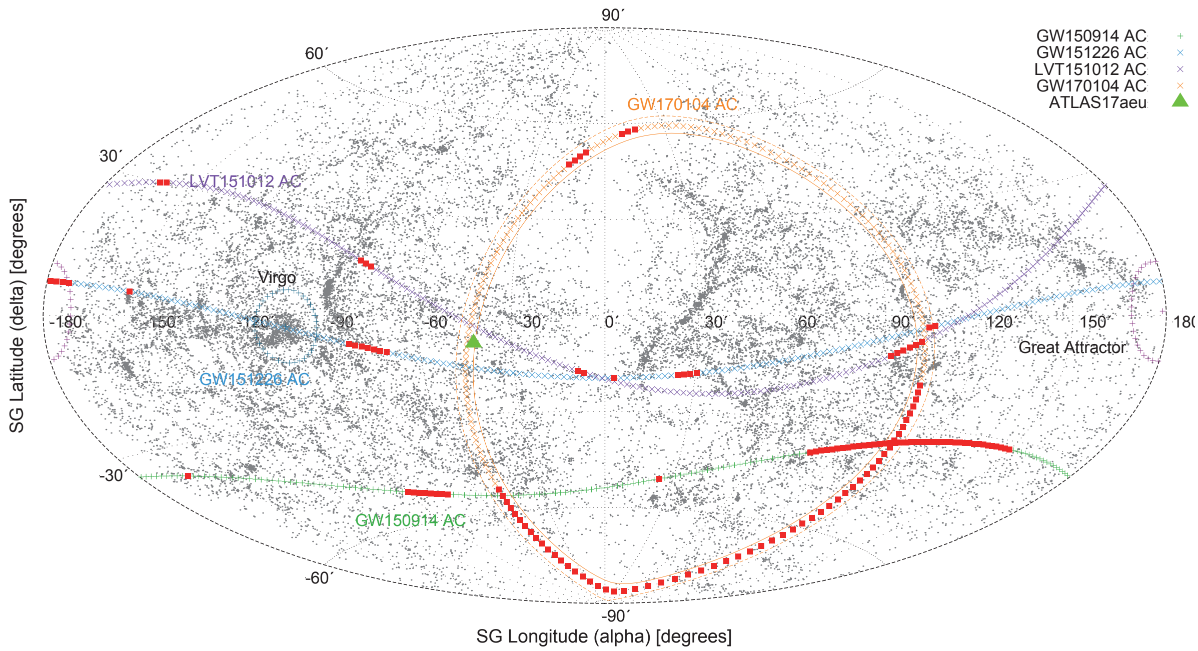


Figure 4: *ACs for LIGO events in supergalactic coordinates. The red points corresponding to the condition $G_L/G_H \approx 1 \pm 10\%$ represent the allowed source positions in the case of tensor "+" ($\Psi = 0$) incoming GW. The green triangle denotes the possible optical counterpart ATLAS17aeu for the GW170104.*

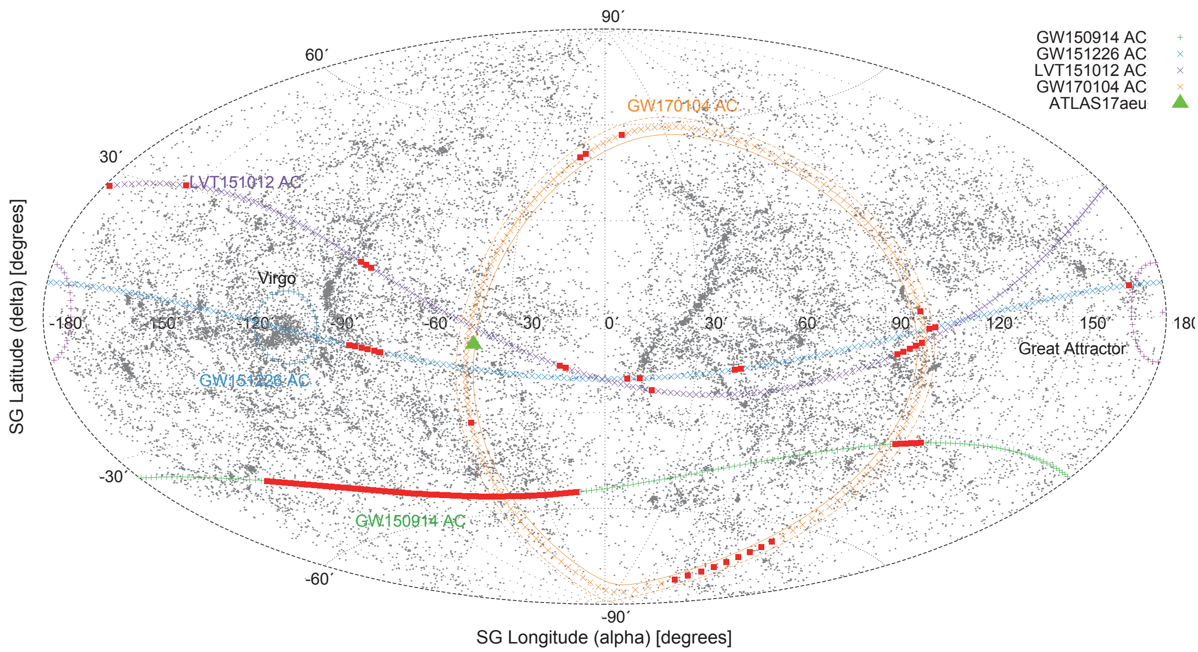


Figure 5*: ACs for LIGO events in supergalactic coordinates. The red points corresponding to the condition $G_L/G_H \approx 1 \pm 10\%$ represent the allowed source positions in the case of scalar (transverse or longitudinal) incoming GW. The green triangle denotes the possible optical counterpart ATLAS17aeu for the GW170104.*

The results of the construction of ACs in SG CS together with the predicted by the method GW source localization (red points) are shown in Fig. (4) for the scalar (longitudinal or transverse) and on the Fig. (5) – for the tensor plus ($\Psi = 0$) polarization of an incoming GW. Interestingly, the ACs for all GW events detected in 2015 lie along the supergalactic plane of

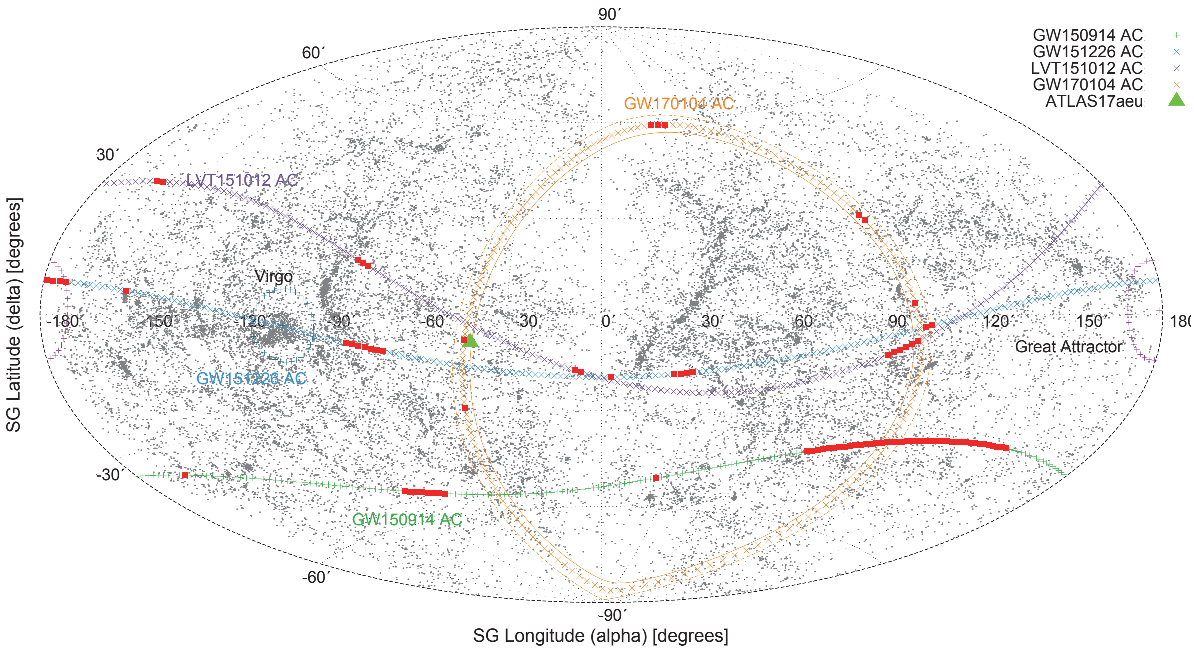


Figure 6: *ACs of the predicted source positions for LIGO events in the case of tensor transverse GW ($G = 1.5F_+ + \sqrt{2}\, F_\times$) in SG coordinates. Red points correspond to the condition $G_L/G_H \approx 1 \pm 10\%$. The green triangle denotes the possible optical counterpart ATLAS17aeu for the GW170104.*

the LSC with a range of possible positions within ±30º SGB. This fact may witness that the possible GW sources are capable to belong to the LSC, i.e. on the distances less than 100 Mpc, which contradicts the currently accepted assumption about these sources as binary coalescences at the distances 400 – 1000 Mpc (Abbott et al. [2016a]). Nevertheless, if such a correlation between the positions of ACs and the SG plane is confirmed by the forthcoming GW observations, it should be also necessary to consider alternative mechanisms of the original GW radiation. For instance, in the context of scalar-tensor metric theories as well as the field gravitation approach, some authors (Novak and Ibanez [2000], Maggiore and Nicolis [2000], Coccia et al. [2004], Maggiore [2006], Baryshev, Yu. V. [1990]) predict existence of a scalar GW radiation from a symmetric core-collapse Supernova (CCSN).

The detected GRB-like afterglow ATLAS17aeu as a possible counterpart event of the GW170104 is indicated by the green triangle. The AC of the GW170104 is depicted taking into account the error in the determination of the time delay ±0.5 ms, Table1, which allows us to represent the realistic possible area for the forthcoming search of electromagnetic transients for this event. Besides this, Fig. (6) presents the case of a mixture of tensor transverse modes with $G = 1.5F_+ + \sqrt{2}F_\times$, where the covering area is coincide with the position of the ATLAS17aeu. Thereby focusing on the fact that there may exist such polarization states of an incoming GW providing various localization areas including those within the SG plane.

6. Conclusions

We have presented the new method for a GW source localization on the sky in the case of a GW detection by two interferometric antennas. The method is based on the antennas beam patterns for a supposed polarization state of the incoming GW together with the measurements of the arrival time delays between antennas and the ratio of the detected strains at each antenna.

It has been shown that a network of LIGO-type two-arm antennas can distinguish between tensor and scalar, but not between scalar longitudinal and transverse polarizations, which is possible by means of one-arm interferometric (as well as bar) detectors.

We have demonstrated that there is an interesting possibility for the polarization state recognition by means of actual localization on the sky a GW source. For this purpose, there has been considered a network of three antennas LIGO-Virgo. The theoretical conclusions concerning beam patterns for different polarization states were applied to the calculations of the strain ratio for each antenna couple in order to offer a test on the polarization state of the GW coming from a definitely located source.

For three aLIGO events: GW150914, GW 151226 and LVT151012, the apparent circles of the allowed GW source positions are parallel to the supergalactic (SG) plane of the Local Super-Cluster of galaxies. Such fact indicates that GW sources of these events might belong to this structure. It is worth noting that if the detected three events did not belong the LSC, then we would have a rare chance of accidentally correlated direction of GW sources positions on the sky, especially for the sources at very high distances such the currently proposed 400 – 1000 Mpc for the LIGO events 2015 (Abbott et al. [2016a], Abbott et al. [2016b]). Moreover, if these GW sources are related to the LSC, then we have to consider distances to them within ~ 100 Mpc.

Interestingly, for the new aLIGO event GW 170104, the apparent circle is perpendicular to the supergalactic equator with only some parts within SG plane (±30º SGB), nevertheless the possible optical counterpart ATLAS17aeu to this event (Stalder et al. [2017]) belongs to the Local Super-Cluster plane, which is also consistent with our supposition about the special role of the LSC.

The next aLIGO observing runs are proposed to test the reality of clustering the GW events along the SG plane. Future identification of GW sources with electromagnetic counterparts is crucial for the physics of the gravitational interaction. Especially, follow-up observations are of great importance for the fundamental physics and they should take into account the experience in GRB optical identification.

Acknowledgements

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