Bull. Spec. Astrophys. Obs., 1998, 45, 117-119

The technique of measuring four Stokes parameters

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Received June 22, 1998; accepted July 10, 1998.

Abstract. A method of analysis of the Stokes parameters is proposed. With quasisimultaneous measurement of the parameters I, Q and U the radiation is passed through the input phase plate $\lambda/4$ (the highest velocity axis makes an angle of 0°), the electro-optical modulator (the highest velocity angle +45°), the polarizer (transmission axis 0°), a voltage corresponding to a phase shift of $+\lambda/4, 0, -\lambda/4$ being fed to the modulator. When analysing the parameters I, Q, and V the input phase plate is positioned so that the highest velocity axis is aligned at an angle of 45°. The radiation is recorded with a panoramic high readout-rate detector, the information being represented in the form of three images, which correspond to the indicated phase shifts of the modulator.

Key words: methods: data analysis – polarization – Stokes parameters

The Stokes parameters characterize the state of radiation polarization. They are symbolized as I, Q, U, V, where I is the intensity, Q, U are the linear polarization parameters, V the circular polarization parameter. Measurement of these parameters provides information of the presence of magnetic field (Zeeman effect). Analysis of circular polarization in spectral lines (V parameter) permits the longitudinal magnetic field strength to be evaluated. Measurements of the Stokes parameters in a continuous spectrum give information on large magnetic fields. The application of up-to-date achromatic plates made it possible to create new generation devices for spectropolarimetric studies (Najdenov and Panchuk, 1996). The common disadvantage of all these devices is that when used with the BTA, the techniques of successive measurement of the Stokes parameters is applied. The patent search has shown that there exist ways of quasisimultaneous measurement of three Stokes parameters. Here are used piezoelastic modulators with sinusoidal modulation (Berdyugin and Shakhovskoi, 1993; Gandorfen and Povel, 1997). When realizing the design with right-angle modulation on KDP-type crystals for the simultaneous measurement of the Stokes parameters, great light losses arise. The devices are bulky, hard to tune and operate. This impedes effective solution of the problems associated with the investigation of polarization in objects whose parameters vary at short times. This is why the introduction of a procedure of fast measuring the Stokes parameters seems quite urgent.

The simultaneous recording of the four Stokes parameters is the technique closest to the proposed procedure in which the radiation is passed through two modulators arranged in an appropriate manner. In turn they are fed a special form voltage (Kuvshinov and Levitan, 1983). Such a device, consisting of two modulators has an essentially sophisticated circuit and decreases the measurement precision because of the enormous light losses. It should be noted that the manufacture of high-voltage generators to control the modulators at a phase shift of $\lambda/2$ is quite a problem.

The aim of the paper is to show that the suggested procedure improves the precision of measurement of polarized radiation parameters. This aim can be attained provided that a minimum number of optical elements is used and a special form modulation is applied.

In the procedure proposed, when measuring concurrently the parameters I, Q and U, the radiation is passed through the input phase plate $\lambda/4$ (the highest velocity axis is at an angle of 0°), the electrooptical modulator (the highest velocity axis is at an angle +45°), polarizer (transmission axis is at 0°), a voltage corresponding to a phase shift of $+\lambda/4, 0, -\lambda/4$ being fed to the modulator. When analysing the parameters I, Q and V, the input phase plate is positioned so that its highest velocity axis is at 45°. The radiation is recorded with a panoramic high readout rate detector, the information being represented as three images which correspond to the indicated phase shifts of the modulator. The Stokes parameters are calculated in the following way:

$$Q = \frac{I_2 - I_1}{I_1 + I_2};$$
$$U = \frac{2I_3 - I_1 - I_2}{I_1 + I_2};$$
$$\Psi = 0.5 \arctan(U/Q);$$
$$V = \frac{2I_3 - I_1 - I_2}{I_1 + I_2},$$

where I_1 is the intensity of the spectrum resolved in the first image, I_2 is that registered in the second image, I_3 in the third one.

The correctness of the method is grounded mathematically by means of Muller matrices. Let M_1 denote the Stokes vector, M_2 the matrix of the input phase plate with the highest velocity axis aligned at the angle of 0°, M_3 the modulator matrix at a phase shift of +45°, M_4 — 0°, M_5 — −45°, M_6 the polaroid matrix, M_7 — the matrix of the phase plate with the highest velocity axis aligned at 45°:

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = M_1$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} = M_2$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = M_3$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = M_4$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} = M_5$$

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} 0.5 = M_6$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = M_7$$

When deriving the Stokes parameters I, Q, U, multiply the matrices for the phase shift KDP $+\lambda/4$, 0, $-\lambda/4$ in the following fashion:

$I_1 = M_6 * M_3 * M_2 * M_1$,
$I_2 = M_6 * M_4 * M_2 * M_1$,
$I_3 = M_6 * M_5 * M_2 * M_1$, to yield

$$I_1 = 0.5 \begin{pmatrix} I - Q \\ I - Q \\ 0 \\ 0 \end{pmatrix}$$

$$I_2 = 0.5 \begin{pmatrix} I + Q \\ I + Q \\ 0 \\ 0 \end{pmatrix}$$

$$I_3 = 0.5 \begin{pmatrix} I + U \\ I + U \\ 0 \\ 0 \end{pmatrix}$$

Calculate the linear polarization parameters by the formulae:

$$Q = \frac{I_2 - I_1}{I_1 + I_2};$$
$$U = \frac{2I_3 - I_1 - I_2}{I_1 + I_2};$$
$$\Psi = 0.5 \arctan(U/Q).$$

To compute the parameters I, Q, V, multiply the matices as follows:
$I_1 = M_6 * M_3 * M_7 * M_1$,
$I_2 = M_6 * M_4 * M_7 * M_1$,
$I_3 = M_6 * M_5 * M_7 * M_1$,
the Stokes parameters are derived from the formulae:

$$Q = \frac{I_2 - I_1}{I_1 + I_2};$$
$$V = \frac{2I_3 - I_1 - I_2}{I_1 + I_2}.$$

The block diagram of the device used to realize the procedure being described is shown in Fig. 1. It incorporates polarized radiation source 1, quarter-wave plate 2, electrooptical modulator 3, Wollastone prism 4, detector 5, distributing unit 6, computer 7, control generator 8.

Fig. 2 displays a facility used to provide the control voltage. It consists of two identical transformers, the secondary windings of which are connected in series; the primary windings are separate. Rectangular pulses shifted by 90°, as shown in Fig. 3, are fed to the primary windings. The transformers symmetrize the voltage at the modulator, which improves the measurement accuracy.

The described procedure has been realized with a two-channel polarimeter the author made for the telescope Zeiss-1000 of SAO. Measurements of astrophysical standards in the spectral bands B and V have

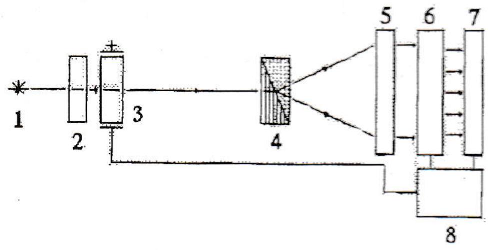

Figure 1: *The block diagram of the device: 1 — polarized radiation source, 2 — plate λ/4, 3 — electrooptical modulator, 4 — polarization dividing unit, 5 — photodetector, 6 — distributing facility, 7 — computer, 8 — control generator.*

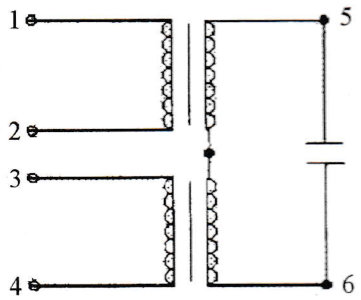

Figure 2: *Connection of the transformer windings.*

shown that accuracy defined by the statistics of photocounts can be attained by using this procedure.

The introduction of the technique at SAO permitted the results published by Beskrovnaya et al. (1995), Somov et al. (1998) to be obtained.

Using the 6 m telescope we can currently study

- polarization variations in spectrum at times of 32 milliseconds (a detector is scanner);

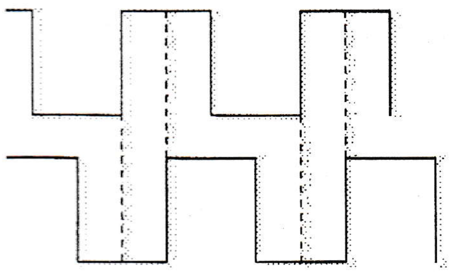

Figure 3: *The shape of pulses fed to the primary windings of the transformers.*

- polarization variations in spectrum from 5 to 10 minutes (CCD detector);

- linear and circular polarization in B and V bands (two-channel polarimeter with a photoelectric multiplier as a detector).

Acknowledgements. The author wishes to express his thanks to N.N. Somov, T.A. Somova, N.V. Borisov, G.G. Valyavin, I.I. Romanyuk for a help in the introduction of the technique, and to V.G. Klochkova and V.E. Panchuk for support in the work.

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