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Study of instrumental polarization at the coude focus of the SAO 1 m telescope

V.D. Bychkov, V.G. Romanenko, L.V. Bychkova

Special Astrophysical Observatory of the Russian AS, Nizhnij Arkhyz 357147, Russia

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Abstract. The paper reports on the study of instrumental polarization in the coude focus of the 1 m telescope of SAO. The value and behaviour of linear instrumental polarization as a function of declination, hour angle and wavelength are evaluated from direct measurements.

Key words: polarimetric measurements – polarization

1. Introduction

One of the most common techniques of stellar magnetic field measurement is based on obtaining Zeeman spectra, i.e. on taking spectra in two circular polarizations with a high spectral resolution (R no lower than 25000). In order to obtain high quality spectra, spectrometers placed at coude focus are mostly used. Unfortunately, they have an essential shortcoming — instrumental polarization due to reflection of the light beam by diagonal coude mirrors before it enters the coude spectrometer. The well known problem has been considered in detail in the papers by Babcock (1959), Wolf and Bonsak (1972), Borra (1976), Borra and Woghan (1977), Nariaj (1982), Capitani et al. (1989) and other authors. The principal difficulties in investigation, taking account and correction of the instrumental polarization consist, on the one hand, in labour-consuming process that depends on a number of factors, on the other hand, such investigations have to be done for each focus of a particular telescope which is used for polarimetry.

2. Observations

For measuring the linear instrumental polarization, the polarimeter MINIPOL was installed at the coude focus of the 1 m telescope. The optical diagram of the coude focus and the location of the polarimeter are displayed in Fig. 1, where the following designations are used: 1 is the main mirror, 2 — the secondary mirror, O3 — the first diagonal 45° mirror, O4 — the second diagonal 45° mirror, O5 — the third diagonal mirror, Pol. — the polarimeter location. The linear polarization measurements were made from polarization standards (Table 1) using the ordinary technique: background — object — etc. The reduction also involved the conventional procedure in which the background contribution is subtracted from the measured radiation of the object.

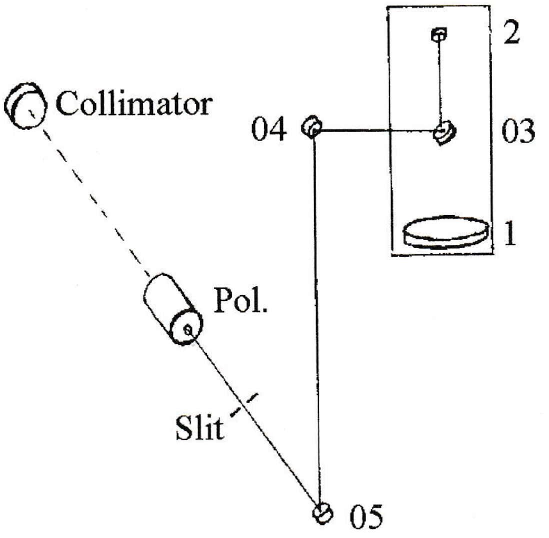

Figure 1: *Polarimeter in the coude focus.*

The linear polarization was measured in the Johnson photometric system UBVRI on September 7—9, 1997 and on March 23, 1998. Actually two parameters Q and U would be measured. The polarization would be found as $P = \sqrt{Q^2 + U^2}$.

For the position angle calibration Glan-Thomson prism was used. As an example, Fig. 2 presents the relationship between the measured linear polarization and hour angle for the star HD 212311 in the V band. The crosses mark the values obtained with an exposure of 1 min. The accuracy of a single measurement ranged from 0.16 to 0.23 %. The sinusoid describes the best fit of instrumental polarization.

Table 1: *The stars used as standards for zero polarization*

HD/BD	α	δ	P(b)	σ	
HD 358	00 05 48	+28 48 52	0.016	0.007	Leroy, 1993
HD 432	00 09 11	+59 08 59	0.015	0.027	Turnsnek, 1990
HD 15318	02 28 10	+08 27 36	0.092	0.024	Turnsnek, 1990
HD 21447	03 30 00	+55 27 07	0.017	0.030	Turnsnek, 1990
HD 103287	11 51 13	+53 58 22	0.013	0.012	Leroy, 1993
BD +32 3739	20 12 02	+32 47 44	0.039	0.021	Turnsnek, 1990
HD 212311	22 21 59	+56 31 53	0.028	0.025	Turnsnek, 1990
HD 214923	22 41 28	+10 41 23	0.028	0.019	Turnsnek, 1990

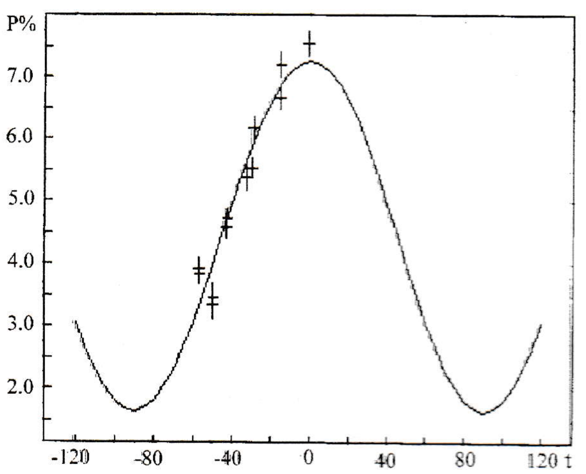

Figure 2: *Instrumental polarization as a function of hour angle for HD 212311.*

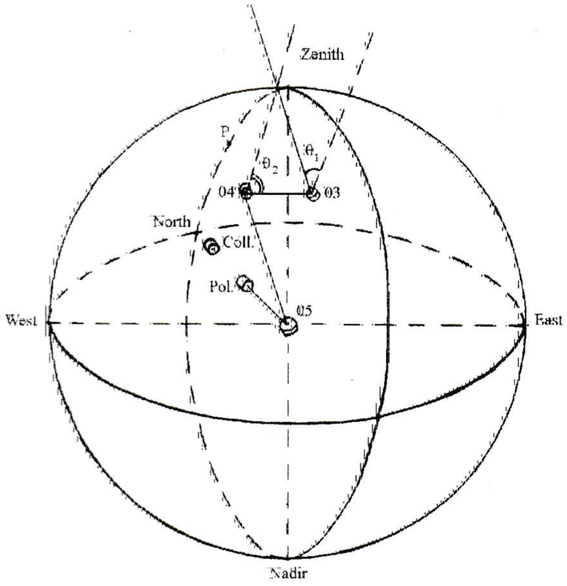

Figure 3: *Orientation of optical axis intercept of coude focus relative to the main directions on the celestial sphere.*

3. Selection of coordinate system

The light beam is sent to the coude in the following manner: the light from the object, having been sequentially reflected from the main and secondary mirrors, strikes diagonal mirrors O3, O4 and O5. The primary and secondary mirrors of the telescope do not introduce appreciable phase distortions into the wave front, since the light is reflected practically by the normal in both cases. The phase shift can be introduced by mirror O3, which is at an angle of 45° to the optical axis and deflects the incident beam by 90° towards axis O5-O4. Mirror O3 is made fast to the telescope tube, i.e. immovable with respect to the main and secondary mirrors, and it rotates about axis O4-O3. The phase shift can also be produced by diagonal mirror O4 positioned at an angle of 45° with the axis O3-O4 and turned about axis O4-O5. The last immovable flat inclined mirror 05 directs the beam to the horizontal plane. The beam of light incident on this mirror forms with it an angle equal to half the latitude of the site and is 21°49′36″. For the sake of convenience in examining the instrumental polarization, it is necessary to choose a coordinate system that could relate the objects' coordinates in the sky to the orientation of the reflecting inclined mirrors. Fig. 3 shows the orientation of the sections of the optical axis with respect to the principal directions on the celestial sphere, which explains the choice of the coordinate system. The direction of light propagation is conventionally assumed to be a positive direction of Z axis. Denote the angle of turn of mirror O3 by Θ_1. The telescope tube direction to the world pole is assumed to be the zero-axis (in this position the surface of mirror O3 is parallel with that of mirror O4). In this case $\Theta_1 = 90° - \delta$. Denote the turn angle about axis O4-O5 as Θ_2. In the case of eastward positioning of the telescope take the eastward direction of axis 04-03 as the origin. Angle Θ_2 will then be related to hour angle t by the expression $\Theta_2 = t - 90°$. In the case of westward repositioning of the telescope $\Theta_2 = t + 90°$.

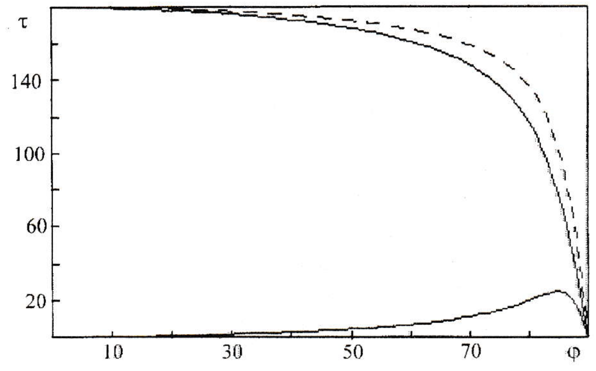

Figure 4: *A comparison of the phase shift, calculated according to Drude (1959) (dashed line) and according to Capitani et al. (1989) (solid line). The line at the bottom shows the difference.*

Reflection from metal-coated mirrors.

As the results of a number of tests (Borra, 1976; Borra and Vaughan, 1977; Capitani et al., 1989) have shown the theoretical estimates of polarization agree well with those obtained in measurements when the mirrors have metal (aluminium) coatings.

The phase shift and the amplitude ratio of the parallel and perpendicularly reflected electric vector of the light wave can be found using the classical approach (Drude, 1959), as it has been done by Borra (1976).

$$\begin{aligned} \mathrm{tg}\,\Delta &= \sin q\,\mathrm{tg}\,2p & (1)\\ \cos 2\chi &= \cos q \sin 2p & (2)\\ \mathrm{tg}\,q &= k & (3)\\ \mathrm{tg}\,p &= \frac{n\sqrt{1+k^2}}{\sin i\,\mathrm{tg}\,i} & (4)\\ \mathrm{tg}^2\chi &= r_p^2/r_s^2, & (5) \end{aligned}$$

where n is the refractive index for optical waves in metal, and k is the absorption coefficient. The value of r_s has been taken from Borra and Vaughan (1977). This is a very simplified scheme in which all high terms are discarded. However, at present, the application of more refined procedures of Capitani et al. (1989) presents no problem. Figs. 4 and 5 show a comparison of the quantities τ and χ as a function of incidence angle, which have been computed from the simplified relationships of Borra (1976) and from more precise formulae in accordance with the metal optics of Capitani et al. (1989).

$$\chi^2 = \frac{f^2+g^2-2f\sin i\,\mathrm{tg}\,i+\sin^2 i\,\mathrm{tg}^2 i}{f^2+g^2 2f\sin i\mathrm{tg}\,i+\sin^2 i\mathrm{tg}^2 i} \qquad (6)$$

$$\mathrm{tg}\tau = \frac{2\,g\sin i\mathrm{tg}\,i}{\sin^2 i\mathrm{tg}^2 i-(f^2+g^2)}, \qquad (7)$$

where

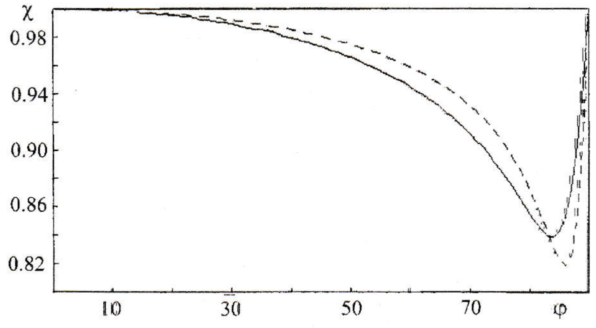

Figure 5: *χ parameter as a function of incidence angle calculated according to Drude (1959) (dashed line) and according to Capitani et al. (1989) (solid line).*

$$\begin{aligned} f^2 &= \frac{1}{2}\left(n^2-k^2-\sin^2 i+E\right), & (8)\\ g^2 &= \frac{1}{2}\left(n^2-k^2+sin^2 i+E\right), & (9) \end{aligned}$$

where (10)

$$E = \sqrt{(n^2-k^2-\sin^2 i)^2+4n^2k^2}.$$

As can be seen from Figs.4 and 5, the phase shift and the ratio of energy coefficients r_s and r_p of the components for an angle of 45°, computed by the two techniques, are markedly different. The computation will further be based on precise expressions (6) – (9). The reflection coefficient for the normal incidence in this case, according to Drude (1959), is

$$R = \frac{(1-n)^2+n^2k^2}{(1+n)^2+n^2k^2}. \qquad (11)$$

The conversion of the parameters of the incident light, when reflected from a flat surface, which is described by the Stokes vector $S=(I,Q,U,V)$, can be written in the matrix form

$$S' = T \times\ S. \qquad (12)$$

Here $S'=(I',Q',U',V')$ is the Stokes vector describing the reflected light parameters; T is the specific case of Muller's matrix for a flat metal-coated mirror obtained by Jager and Oitken (1963a and 1963b), Capitani et al. (1983).

$$T = \frac{r_s^2}{2}\begin{pmatrix} X^2+1 & X^2-1 & 0 & 0\\ X^2-1 & X^2+1 & 0 & 0\\ 0 & 0 & 2X\cos\tau & 2X\sin\tau\\ 0 & 0 & -2X\sin\tau & 2X\cos\tau \end{pmatrix}, (13)$$

where $X^2=r_p^2/r_s^2$ is the ratio of the Fresnel amplitude coefficients of the electric vector parallel with and perpendicular to the plane of incidence. τ is the difference in phase shift between reflections p and s. The matrix of turn through angle Θ, according to Shercliff (1962), has the form

$$R(\Theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\Theta & \sin 2\Theta & 0 \\ 0 & \sin 2\Theta & \cos 2\Theta & 0 \\ 1 & 0 & 0 & 0. \end{pmatrix} \tag{14}$$

Then basing on (13) we can write the expression of the vector $S(I,Q,U,V)$ in a general form:

$$S' = T_5 \times R(\Theta_2) \times T_4 \times R(\Theta_1) \times T_3 \times S \tag{15}$$

The result of multiplying together (following the rules of matrix algebra) the written matrices leads to very complicated expressions, and only application of computer programmes will make it possible to effectively solve these problems in a numerical form. Since in our case we have no direct measures of n and k, in the calculation we will make use of the values derived by Schultz (1954) and Schultz and Tangherlini (1954) for aluminized mirrors, $n = 1.113$ and $k = 6.39$. Values close to these estimates, for aluminium-coated astronomical mirrors, have been obtained by Capitani et al. (1989) — $n = 1.036 \pm 0.004$ and $k = 5.89 \pm 0.01$. So far as in our case the diagonal coude mirrors are manufactured by the same firm, at the same time, and using the same technology, they may be assumed to have the same reflectance, i.e. close n and k values. Taking as an example n and k derived by Schultz and Tangherlini (1954) and Capitani et al. (1969), we have computed the instrumental polarization depending on hour angle, using expressions (6) — (14), for the case of HD 212311. When calculating the Stokes output parameters for the input vector we adopted

$$S(I,Q,U,V) = S(1,0,0,0) \tag{16}$$

since, as can be seen from Table 1, there is no polarization in the radiation from standard stars. The results are presented graphically in Fig. 6. The upper solid curve is plotted using the n and k values from Capitani et al. (1989), the dashed line is the n and k values of Schultz and Tangherlini (1954). The crosses indicate the measured polarization values in the V band. As can be well seen from Fig. 6, the computed values with the adopted values of n and k are essentially higher than the observed ones. The lower solid curve is a sinusoid fitted with the estimates made. Fig. 7 displays the polarization assessed from the observations of the star HD 214923 versus hour angle. In accordance with theory the derived relationships are close to sinusoidal with a maximum in the meridian (t=0). It should be emphasized that in Figs. 6 and 7 are presented, as an example, the typical relationships between instrumental polarization and exactly hour angle, which can be derived with relative ease by observing at different t even without repointing towards different objects.

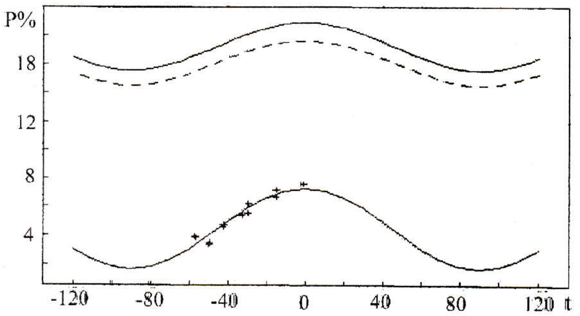

Figure 6: *Calculated and observed instrumental polarization. Solid line (top) according to Capitani et al. (1989). Dashed line according to Schultz and Tangherlini (1959). Crosses are the observed values.*

It is much more difficult to study the behaviour of the instrumental polarization as dependent on the declination. This will require quite a few polarization standards at different declinations (see Table 1). In Fig. 8 linear instrumental polarization is shown as a function of declination in the meridian for the U and V bands. The accuracy of measurements is represented by the bars and ranges from 0.04 % to 0.22 %, but for the star BD +32°3739, for which the polarization estimate in U is about 1 %. The upper curve is the relation computed with n and k obtained by Capitani et al. (1989) for aluminium-coated mirrors. The crosses indicate the observed values. Because the instrumental polarization behaviour fails to be described by using n and k derived in other papers, it was attempted to find the values of these parameters to satisfy best the whole collection of the measurements. The search was done by suitable selection of n values in the range from 0.05 to 3.0 and k values from 1.0 to 30.0, assuming the parameters to be equal for the three mirrors. In so doing, those values were chosen that set an upper limit on the observed polarizations. The values found are $n = 1.00$, $k = 9.5$ and known to be lower than the values computed, using those ones. The reflection coefficient with such n and k, in accordance with expression (10) is $R = 0.81$, which is real enough for our case.

4. The linear instrumental polarization behaviour

As noted above, the instrumental polarization measurements were specially made in the UBVRI bands of Johnson system. This made it possible to assess the instrumental polarization behaviour with wavelength. Since the polarization varied with hour angle, and the measurements were made sequentially in all the bands (background — object), to derive such a relationship, we had to reduce the polarization measures by one hour angle corresponding to the middle of the measuring cycle. The cycle generally was for

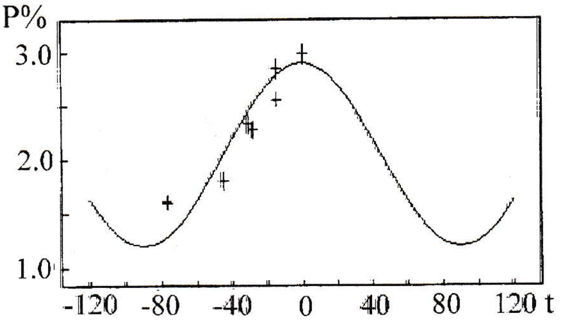

Figure 7: *Instrumental polarization as a function of hour angle for HD 214923.*

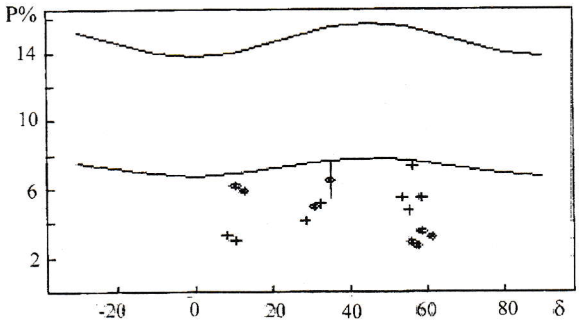

Figure 8: *Instrumental polarization as a function of declination in meridian for U and V bands.*

about 20–30 minutes, which makes 5–7 degrees in the hour angle measures. Fig. 9 displays the mean curve, where the polarization is shown as a function of wavelength for four cycles of measurements in HD 212311. The crosses and bars, corresponding to the standard error values, indicate the polarization estimates, and the mean values have been used to plot the relationship. The filters used and the mean polarization values are listed in Table 2.

As those data show, the linear polarization tends to grow with wavelength by an average of 0.47 % per each 1000 Å; this, however, is characteristic of large declinations, $\delta > 50°$. At smaller declinations in the violet range, in the U band, the instrumental polarization behaves in a different manner as compared to the rest of the spectral intervals: it begins to rise. In Fig. 10 the instrumental polarization, derived for the star HD 214923 over 7 cycles, is shown against wavelength. The crosses indicate individual values; the curve is drawn through the mean values listed in Table 2. Fig. 11 shows the instrumental polarization measurement in meridian position of the telescope depending on declination (in the U and V bands). The straight line shows a possible linear regression when describing these relationships, while the dashed curve is the theoretical relation computed with found n and k.

Fig. 12a-d shows the theoretical relations between the Stokes parameters Q, U, V and polarization and hour angle. The relations have been computed for different declinations with a step of 15° in an interval from –30° to 90°. It will be recalled that according to the above-said they give the upper instrumental polarization estimates.

Table 2: *Measured polarization of stars of different declination angles*

		HD212311		HD214923	
Filter	Centre	P(%)	σ(P)	P(%)	σ(P)
U	3550	3.545	0.120	5.896	0.219
B	4650	4.329	0.228	2.432	0.281
V	5500	4.436	0.527	2.374	0.155
R	6100	4.457	0.332	2.589	0.185
I	8100	5.690	0.167	3.670	0.106

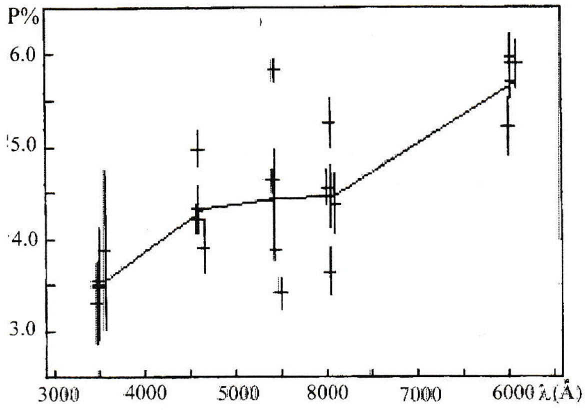

Figure 9: *Linear polarization as a function of wavelength for HD 212311.*

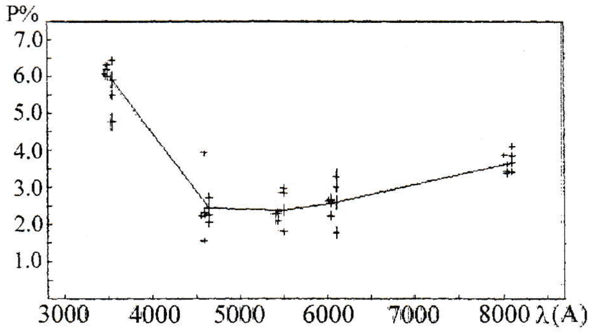

Figure 10: *Linear instrumental polarization as a function of wavelength for HD 214923 located near the equator.*

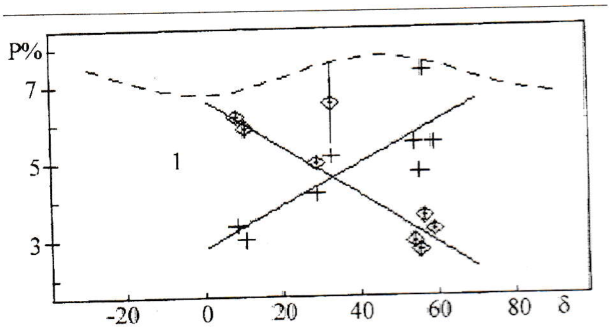

Figure 11: *Linear relationship between instrumental polarization and declination for U and V bands. Theoretical relation is shown by the dashed line.*

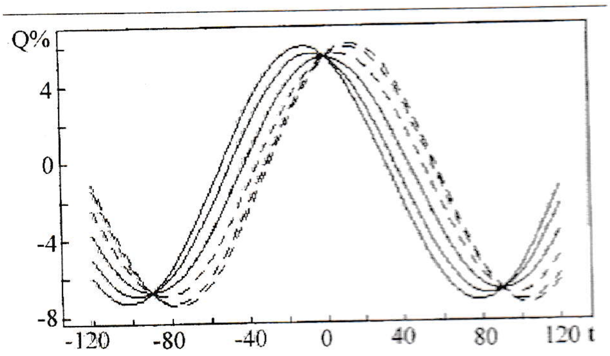

Figure 12: *a-d. The theoretical values of Q, U, and V parameters and polarization as a function of declination and hour angle.*

5. Discussion

As it was stated by Severny et al. (1974) the parameters n and k determined by different procedures and authors have quite a wide range of values, which can be explained by the difference in structure and thickness of the aluminium coating of mirrors, methods of its application and treatment, deterioration with time, etc.

According to the tests conducted by Borra (1976), the reflecting properties of mirrors change during the first few months (deterioration), and after that they remain practically unchanged for a very long time. It is well known from physics (from metal optics, in particular), the reflection of electromagnetic wave is different at thin and thick metal coatings, and, moreover, at that plated with a thin layer of dielectric. In our case we have a reflection from a thin flat-parallel aluminium film, interposed between two dielectric media, one of which (glass bulk of the mirror) is infinite, the other (protective layer) is a thin flat-parallel layer of dielectric. Namely, the aluminium reflecting coating is protected from outside action by a transparent magnesium fluorite (see telescope technical description of the manufacturer "Zeiss"). According to Born and Wolf (1964) any phase shift can be attained in this case. All depends on the thickness, structure, age and other parameters of the layers involved in reflection. That is, the correct computation of the phase shift is very complicated because some parameters of mirrors are unknown and one has to rely upon direct measurements. In this paper we present quantitative study of the linear instrumental polarization versus the telescope position and in dependence on wavelength.

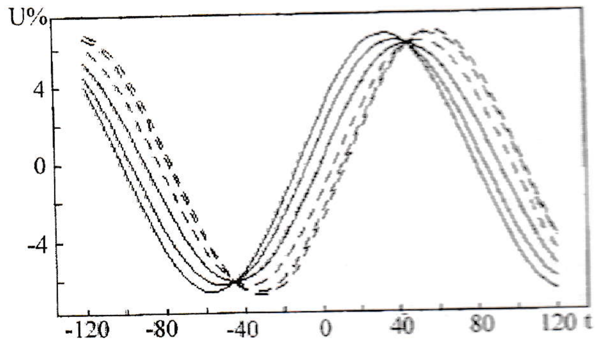

Figure 12: *b.*

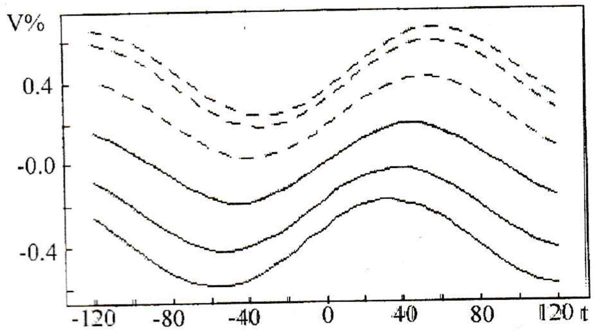

Figure 12: *c.*

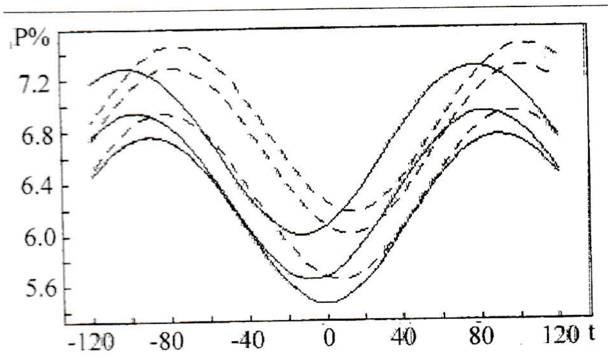

Figure 12: *d.*

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References

Babcock H.W., 1958, Astrophys. J. Suppl. Ser., **3**, 141

Borra E.F., 1976, Publ. Astr. Soc. Pacific, **88**, 548

Borra E.R. and Vaughan A.H., 1977, Astrophys. J., **216**, 462

Borra F.E., Landstreet J.D., Thompson I., 1983, Astrophys. J. Suppl. Ser., **53**, 151

Drude P., 1935, Optics, ONTI, L.-M., 468

Drude P., 1959, "The Theory of Optics", Dover Publ.Inc

Capitani C., Cavallini F., Ceppatelli G., Landi Degl'Innocenti M., Landolfi M., Righini A., 1989, Solar Physics, **120**, 173

Jager F.W., Oetken L., 1963, Publ. of Astrophys. Obs. (Potsdam), **31**, 1, No. 103, 1

Jager F.W., Oetken L., 1968, **31**, 5, No. 107, 1

Leroy J.L., 1993, Astron. Astrophys. Suppl. Ser., **101**, 551

Nariai K., 1982, Annals Tokyo Astr.Obser.S.S., **19**, 55

Severny A.B., Kuvshinov V.M., Nikulin N.S., 1974, Izvestiya KrAO, **50**, 3

Snezhko L.I., 1997, private communication

Schultz L.G., 1954, J.Opt.Soc.America, **44**, 357

Schultz L.G. and Tangherlini F.R., 1954, J. Opt. Soc. America, **44**, 362

Schurcliff W.A., 1962, Polarized Light, Harvard Univ.Press, Cambridge, Massachusetts

Turnsnek D.A., Bonhlin R.C., Williamson R.L., Lupie O.L., Koordneef J., Morgan D.N., 1990, Astron. J., **99**, 1243

Wolff S.C. and Bonsack W.K., 1972, Astrophys. J. Suppl. Ser., **176**, 425