Bull. Spec. Astrophys. Obs., 1998, 44, 105–109

Determination of the internal structure constants of the components of the WR type eclipsing binary CQ Cep

T. A. Kartasheva[a], **M. A. Svechnikov**[b]

[a] Special Astrophysical Observatory of the Russian AS, Nizhnij Arkhyz 357147, Russia
[b] Ural State University, Ekaterinburg 620083, Lenina 51

Received December 18, 1997; accepted January 22, 1998.

Abstract. The apsidal motion detected in the WR type eclipsing binary CQ Cep is used to determine constants k_{2WR} and k_{2O} of the internal structure of the system's components, which characterize the degree of concentration of matter towards the centres of the stars. Calculations have shown that $k_{2WR} \approx k_{2O} \approx \overline{k}_2 = 0.0003$, which is nearly an order of magnitude less than k_2 values obtained for main sequence stars. A comparison is made of this result with the one obtained for close binary systems (α Vir, V380 Cyg, δ Ori(A), V1765 Cyg and β Per) that contain a far evolved component. The value of the constants of the internal structure of CQ Cep components is noted to be close to those of $\overline{k}_2$ for δ Ori(A) and V1765 Cyg, the most evolved systems with the apsidal motion. A direct comparison of results obtained for CQ Cep, as well as for the other five far evolved systems, with that given by theory, seems impossible as yet since special model calculations for each of those systems are needed.

Key words: binaries: eclipsing – stars: constants – stars: Wolf–Rayet – stars: individual (CQ Cep)

The study of the orbital period behaviour of the most close of WR binaries ($A = 20\,R_\odot$) CQ Cep has already a semi–centenial history described in detail in our earlier paper (Kartasheva, Svechnikov, 1989). The latest version of interpretation of the (O–C) diagram is available in the thesis of one of the authors (Kartasheva, 1995), which suggests that probability of existence of a third body in the system (a star with the mass $M \approx 17.5 M_\odot$) is very high.

Svechnikov (1954) noted that oscillations of a smaller amplitude, in which the primary and secondary minima displace in opposite directions, are superimposed on the main run of the (O − C$_1$) and (O − C$_2$) diagrams of CQ Cep (diagrams of departures of the observed primary and secondary minima from the calculated ones). This might be evidence of apsidal motion in the system.

Kurochkin (1979) also paid attention to the presence of a slight cyclic oscillation with a period of 54 years on the system (O–C$_1$) diagram.

Being aware that a so far evolved system may have but a small orbit eccentricity (not clearly observed spectroscopically) we have turned to examination of the difference (O–C$_2$)-(O–C$_1$) diagram of CQ Cep (Kartasheva, Svechnikov, 1988, 1989). The latter attracted us by being free from both the deviations associated with direct change of the orbital period and the deviations that arise in the case the system contains a third body. The most important in this diagram is the doubling of the amplitude of the effect produced by the apsidal motion, which facilitates its detection. Indeed, as has been shown by Kartasheva and Svechnikov (1989),

$$(O - C_2) - (O - C_1) = 2b\cos(\omega_{02} + \dot{\omega}_2 E),$$

where
$b = \frac{Pe}{\pi}$ (e is the orbital eccentricity, P is the orbital period of the system),
ω_{02} is the periastron longitude for the epoch T_0,
$\dot{\omega}_2$ is the observed rate of the periastron longitude change, ($U = \frac{360^\circ}{\dot{\omega}_2}$ is the apsidal rotation period),
E is the number of the cycles from the moment T_0.

The possibility of representation of the CQ Cep (O–C$_2$)-(O–C$_1$) diagram by the inclined cosinusoid

$$(O - C_2) - (O - C_1) = 0\overset{d}{.}034 - -0\overset{d}{.}0000029E + 0\overset{d}{.}020\cos(295^\circ + 0\overset{\circ}{.}03E) \quad (1)$$

(see Fig. 1) has allowed us to conclude that the apsidal motion superimposing on the linear run of the (O–C$_2$)-(O–C$_1$) differences does occur in the system. The linear run is, probably, associated with the different and time–variable degree of distortion of the primary and secondary minima by the circumstellar gaseous medium (Kartasheva, 1995).

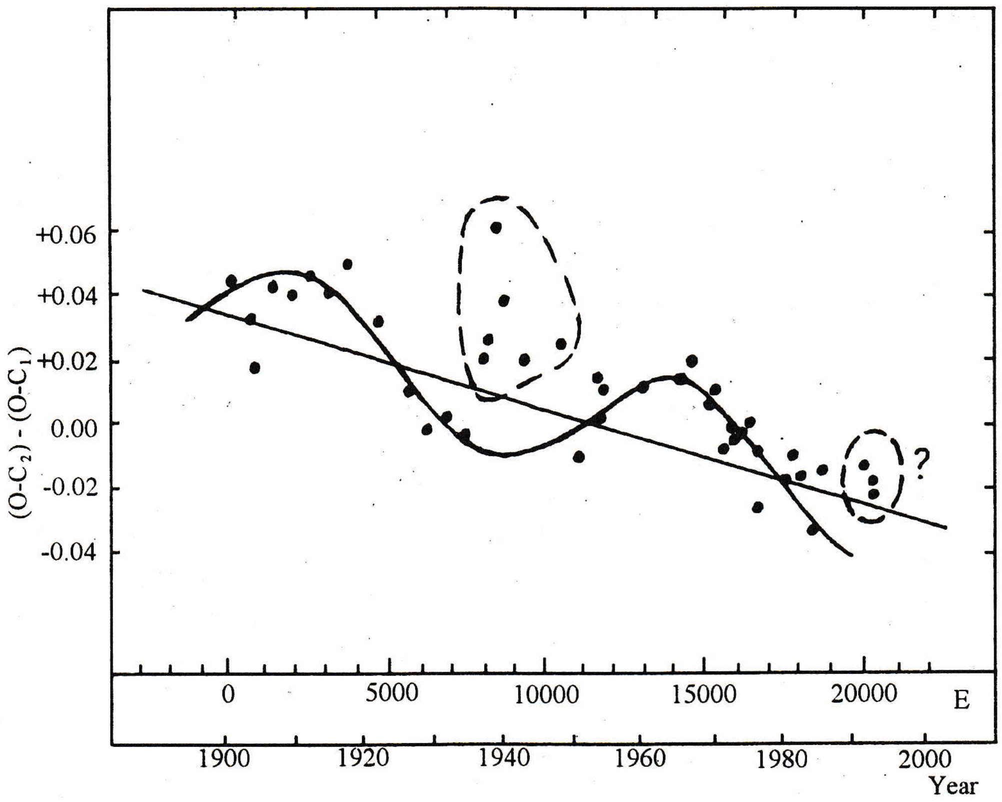

Figure 1: *Difference (O–C$_2$)-(O–C$_1$) diagram of CQ Cep. The dashed line outlines the anomalous regions.*

The representation of the difference diagram by formula (1) permitted the following characteristics of the system's orbit to be derived: e = 0.02; $\dot{\omega}_2$ = $0^{\circ}\!.03$ per a period ($6^{\circ}\!.67$ per a year); U = 54 years; U/P = 12000.

When interpreting the difference diagram of CQ Cep we rejected the (O–C$_2$)-(O–C$_1$) values referring to the period 1936–1949 which we considered as anomalous on the basis that photometric and spectral observations related to it suggested high-velocity ejections of matter from the system (for details see Kartasheva, Svechnikov, 1988). The new moments of the minima of CQ Cep obtained by Kilinc in 1991–1992 (Kilinc, 1994), that is, spaced from the beginning of the anomalous period by a time interval close to the period of apsidal rotation (U), were of interest from the point of view of the behaviour of the (O–C$_2$)-(O–C$_1$) differences. The new values on the (O–C$_2$)-(O–C$_1$) diagram (see Fig. 1) showed again an upward departure of points (by ≈ 0.03^d) from the inclined cosinusoid. Whether a new anomalous region appeared on the (O–C$_2$)-(O–C$_1$) difference diagram of the system, which might suggest the relation of this phenomenon with the apsidal motion, or the linear run of the (O–C$_2$)-(O–C$_1$) differences associated with the distortion of the minima by the interstellar gaseous medium changed its character, is likely to be stated with assurance in two–three decades. So far we have attempted to use the revealed apsidal motion for the determination of the constants of the internal structure of the system's stars, hoping that the estimates made will prove that our interpretation of (O–C$_2$)-(O–C$_1$) difference diagram is correct.

There are three basic reasons for the apsidal motion in a binary system with an elliptical orbit (Batten, 1973):

1. Tidal interaction and axial rotation of the pair components. The rate of the apsidal rotation ($\dot{\omega}_{class}$) due to those effects is described by the following equation (Martynov, 1971):

$$\begin{aligned}\dot{\omega}_{class} = \frac{360}{P}\Bigg\{ & k_{21}r_1^5\left[\frac{m_2}{m_1}15f_2(e)+\left(\frac{\omega_{r1}}{\omega_k}\right)^2\left(1+\frac{m_2}{m_1}\right)\frac{1}{(1-e^2)^2}\right]+\\ & +k_{22}r_2^5\left[\frac{m_1}{m_2}15f_2(e)+\left(\frac{\omega_{r2}}{\omega_k}\right)^2\left(1+\frac{m_1}{m_2}\right)\frac{1}{(1-e^2)^2}\right]\Bigg\} =\\ & =\frac{360}{P}(k_{21}c_1+k_{22}c_2)(\text{degr/year}),\end{aligned} \qquad (2)$$

where

k_{21} and k_{22} are the internal structure constants of the stars of the system,
r_1 and r_2 are the relative radii of the stars expressed in fraction of the system's orbit radius,
m_1 and m_2 are the masses of the components in solar masses,
P is the orbital period of the system in years,
e is the orbit eccentricity,
$f_2(e) = (1-e^2)^{-5}(1+\frac{3e^2}{2}+\frac{e^4}{8})$,
$\frac{\omega_{ri}}{\omega_k}$ is the ratio of the angular velocity of the axial rotation of the i component to that of the orbital rotation,
c_1 and c_2 are the coefficients at k_{21} and k_{22}, adopted further as the weights of the latter.

According to the statistical studies by Swings (1936)

$$\frac{\omega_r}{\omega_k} = \frac{1+e}{1-e}.$$

Formula (2) allows the weighted average value of the internal structure constant of the system's stars to be determined

$$\bar{k}_2 = \frac{k_{21}c_1 + k_{22}c_2}{c_1+c_2} = \frac{\dot{\omega}_{class}P}{360(c_1+c_2)}. \qquad (3)$$

2. Relativistic periastron motion (the effect noticeable in massive systems). The velocity of the apsidal rotation caused by the relativistic effect is determined by the formula that follows from the expression for period of the relativistic apsidal rotation (Batten, 1973):

$$\dot{\omega}_{rel} = 0.64 \cdot 10^{-5}\frac{360(m_1+m_2)}{PA(1-e^2)}(degr/year),$$

where A is the major semi-axis of the system's orbit in the solar radii, the rest designations are the same, as in the item 1.

3. Presence of a third body disturbing the orbit of a binary system. The apsidal motion velocity is determined in this case by the equation following from the expression for the period of the apsidal rotation occurring under the action of the third body (Batten, 1973):

$$\dot{\omega}_{th-dbody} = 0.75\frac{360m_1(P/P')}{P'(m_1+m_2+m_3)}(degr/year),$$

where
P' is the orbital period of a wide system in years,
m_3 is the third body mass in solar masses,
the other designations are given in 1.

In the case of CQ Cep the three causes of the apsidal motions are possible and hence

$$\dot{\omega}_{obs} = \dot{\omega}_{class} + \dot{\omega}_{rel} + \dot{\omega}_{thd-body} = 6.67(degr/year).$$

Since the purpose of our paper was the determination of the internal structure constants of the system's stars, which characterize the degree of concentration of matter towards the star's centres, our next step was to clean the observed velocity of the apsidal motion from the component connected with the relativistic effect and the third body presence. The principal parameters of the system needed for further calculations were taken from Kartasheva and Svechnikov (1989, 1996) and presented in the bottom line of Table 1. Besides, we used the estimate of the period of a wide system ($P' \approx 120$ years) and of the third body mass ($M \approx 17.5 M_\odot$) obtained by Kartasheva (1995).

The calculations performed yielded for the velocity of the apsidal motion caused by the relativistic effects

$$\dot{\omega}_{rel} = 1.04(degr/year)$$

and for the velocity defined by the third body effect

$$\dot{\omega}_{thd-body} = 0.00003(degr/year).$$

Hence it follows that the velocity defined by the tidal interaction and axial rotation of the components of the pair

$$\dot{\omega}_{class} = 6.67 - 1.04 = 5.63(degr/year).$$

The results obtained permitted us to conclude that the part played by the third body in the observed apsidal motion is minor and 80 % of this motion is caused by classical effects.

The weighted mean value of the constant of the internal structure of the system's stars ($\bar{k}_2$) was further estimated by formula (3) and turned out to be 0.0003. An attempt to pass from the weighted mean value of $\bar{k}_2$ for the stars of the system to the internal structure constant of WR star, using formula (3) and assuming a companion to refer to main sequence stars, resulted in negative k_{2WR} value. Return to the real positive k_{2WR} value, as calculations had shown, was possible only provided that the constant of the internal structure of the O-component (k_{2O}) was very close in value to $\bar{k}_2$ and, therefore, to k_{2WR} as well ($k_{2O} < 0.00035$). The latter confirms the conclusions drawn by Marchenko et al. (1995), Kartasheva and Svechnikov (1996) and Kartasheva (1996) that the O-component of CQ Cep is likely to be a far evolved star, though its dimension and physical characteristics (Kartasheva and Svechnikov, 1996) seem small for a bright giant or supergiant.

The result for CQ Cep should have further been compared with the one given by binary systems that contain a far evolved component. As a result of looking through of all the literature available to us five binary systems have been selected which contain a component of luminosity classes IV, III, II, Ib and show a reliable apsidal motion. Together with CQ Cep they form a group of binaries including:

Table 1:

Star	P	e	$\dot{\omega}_{obs}$ $\left(\frac{degr}{year}\right)$	m_1 $(m_\odot)$	m_2 $(m_\odot)$	A $(R_\odot)$	r_1	r_2	$\overline{k}_2$ (log $\overline{k}_2$)	Ref
β Per (B8V+G4–5IV)	2.87	0.01	11.25	3.8	0.82	14.1	0.204	0.251	0.0031 (-2.51)	(1),(2)
α Vir (B1III–IV+B2–3V)	4.0145	0.13	2.52	10.9	6.8	27.5	0.291	0.160	0.0023 (-2.64)	(1),(3),(4)
V380 Cyg (B1.5II–III+B2–3V)	12.4257	0.22	0.26	12.1	7.3	60.6	0.267	0.068	0.0010 (-3.00)	(1),(5)
δ Ori(A) (O9.5II+B1IV)	5.7325	0.06	1.71	23.0	9.0	43.0	0.400	0.240	0.0006 (-3.22)	(1),(6)
V1765 Cyg (B0.5Ib+B2V)	13.3738	0.33	0.17	25.0	12.2	79.1	0.268	0.075	0.0004 (-3.39)	(1),(7)
CQ Cep (WN7+O9.5III)	1.6412	0.02	6.67	22.0	18.3	20.1	0.424	0.283	0.0003 (-3.52)	(8),(9)

(1)	Svechnikov, 1986
(2)	Ho-IlKim, 1989
(3)	Lyubimkov et al., 1995
(4)	Dukes, 1974
(5)	Lyubimkov et al., 1996
(6)	Koch and Hrivnak, 1981
(7)	Mayer et al., 1991
(8)	Kartasheva and Svechnikov, 1989
(9)	Kartasheva and Svechnikov, 1996

- four separated systems with far evolved more massive primary components (α Vir, V380 Cyg, δ Ori(A), V1765 Cyg);
- half-separated system, whose component underwent "a change of roles", having now less massive companion–subgiant (β Per);[1]
- a practically contact system (CQ Cep), whose components underwent "a change of roles", as a result of which a more evolutionary advanced WR star turned out to be less massive. Besides, the O-component of the system, which most likely belongs to luminosity class III (Kartasheva and Svechnikov, 1996; Kartasheva, 1996), has also essentially evolved rightward from the initial main sequence.

For all the systems selected for the comparison with CQ Cep the most reliable mass and relative radii estimates of the stars as well as the latest estimates of parameters of their elliptical orbits have been found. All the data are collected in Table 1. Using the methods described above, for the selected stars the weighted mean values of the apsidal motion constants ($\overline{k}_2$) listed in the last column of Table 1 have been determined. In the last column are presented the references to the sources from which the data on the characteristics of the systems and their component stars have been taken. Since for the first five systems of Table 1 the contribution of the internal structure constants of the evolutionary less advanced components to the weighted mean $\overline{k}_2$ value is small (because of their small sizes and strong dependence of the k_2 values on the sizes of the stars ($k_2 \propto r^5$)), the estimates of $\overline{k}_2$ actually characterize the degree of concentration towards the centres of the stars for the more evolutionary advanced components of the systems. It is seen from the table that for the stars of our sample the internal structure constants either correspond to lower estimates of $\overline{k}_2$ obtained for main sequence stars ($\overline{k}_2 = 0.013 \div 0.0035$; Batten, 1973, Claret and Gimenez, 1993) or are 2 ÷ 8 times as small. The value of $\overline{k}_2$, we have derived for CQ Cep, has proved to be close to those found for δ Ori(A) and V1765 Cyg — the most evolutionary advanced from the systems being compared. This suggests that the result obtained for CQ Cep to be reliable and therefore confirms our interpretation of the $(O - C_2) - (O - C_1)$ diagram of CQ Cep.

As to the agreement of the observed $\overline{k}_2$ values of Table 1 with the model calculations, three out of six stars of our sample (α Vir, V380 Cyg and V1765 Cyg) were included into the theoretical study that have recently been carried out by Claret and Gimenez

---

[1] Review of the new literature on another two half-separated subgiants (W Del and TX UMa) selected by Batten (1973) and Cisneros–Parra (1970) has shown that the latest examination of the behaviour of their orbital periods (Douglas, 1973; Kreiner and Tremko, 1980) has not confirmed the existence of the apsidal motion expected in them. However a close scrutiny of the last (O–C) diagram of TX UMa (Komzik et al., 1992) allows a longer period (U≈80 years) of cyclic variations of the (O–C) differences to be suspected in this system. Whether the variations are brought about by the apsidal motion or suggest a third body to be present in the system is to be decided from observations of the moments of the secondary minimum

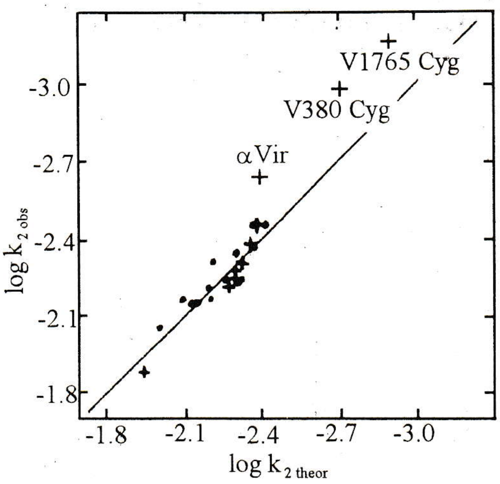

Figure 2: *The observed log k_2 values vs the theoretically predicted.*

(1993). It is for these three stars (see Fig. 2, which is a reproduction of Fig. 13 from the paper of those authors) that the model calculations have still yielded the lower degree of concentration of matter towards the stars' centres (larger $\overline{k}_2$) as compared with what follows from observations. This result is likely to suggest that the model likening of a close binary system to two independently developing stars is invalid not only for half-separated and contact systems but also for far evolved separated pairs. In this connection the paper by Cisneros-Parra (1970) is significant. He showed in his calculation of evolution of close binary systems, including also the phases of mass exchange, how effective the process of mass exchange in close binary systems is for decreasing the constants of the internal structure of the stars. Thus a direct comparison of the Table 1 results (and, in particular, the result for CQ Cep) with that given by theory is impossible yet, since special model calculations are probably needed for each system.

In conclusion it would be worthwhile to touch upon the matter of existence of eccentrical orbits in far evolved systems. From statistical studies we see that the loss of mass and the process of "change of roles" (alongside the tidal interaction) facilitates the reduction of the initial orbital eccentricity of the system. Nearly all far evolved systems (if they have had no supernova outburst) have zero or very slight eccentricities. Though with non-symmetric release of the envelope (and in general with non-symmetric loss of matter) a slight eccentricity may appear again. This is probably the case with CQ Cep. Algol (β Per) provides an example of existence of a minor eccentricity in a half-separated binary system. It is possible that other half-separated systems and the systems close to contact systems have slight eccentricities which are difficult to detect. The cause is both the small depth of the secondary minimum and the complex shape of the (O–C) diagram caused both by complicated change of the period and by distortions of moments of the minima by the interstellar gaseous medium.

References

Batten A.H.: 1973, Binary and multiple systems of stars, Pergamon Press Oxford New York. Toronto, 323

Cisneros-Parra J.U.: 1970, Astron. Astrophys., **8**, 141

Claret A., Gimenez A.: 1993, Astron. Astrophys., **277**, 487

Douglas S.: 1973, J. Amer. Assoc. Var. Star. Observ., **2**, 20

Dukes R.J.: 1974, Astrophys. J., **192**, 81

Ho-IlKim: 1989, Astrophys. J., **342**, 1061

Kartasheva T.A.: 1995, Thesis, Nizhnij Arkhyz

Kartasheva T.A.: 1996, Bull. Spec. Astrophys. Obs., **39**, 78

Kartasheva T.A., Svechnikov M.A.: 1988, Stars of Wolf-Rayet type and relative objects, Tallin, 126

Kartasheva T.A., Svechnikov M.A.: 1989, Astrofiz. Issled. (Izv. SAO), **28**, 3

Kartasheva T.A., Svechnikov M.A: 1996, Bull. Spec. Astrophys. Obs., **39**, 66

Kartasheva T.A., Snezhko L.I.: 1985, Bul. Abastum. Astrofiz. Obs., **58**, 25

Kilinc B.: 1994, Inform. Bull. Var. Stars, 3998

Koch R.H. and Hrivnak B.J.: 1981, Astrophys. J., **248**, 249

Komzik R., Hric L., Grygar J.: 1992, Astrophys. Space Sci., **198**, 149

Kreiner J.M.,Tremko J.: 1980, Bull. Astron. Inst. Czech., **31**, 343

Kurochkin N.E.: 1979, Astron. Circ., **1063**, 1

Martynov D.Ya.: 1971, Eclipsing variable stars, ed. Tsesevich V.P. (in Russian), Nauka, Moscow, 350

Lyubimkov L.S., Rachkovskaya T.M., Rostopchin S.I., Tarasov A.E.: 1995, Astron. Zh., **72**, 212

Lyubimkov L.S., Rachkovskaya T.M., Rostopchin S.I., Tarasov A.E.: 1996, Astron. Zh., **73**, 55

Marchenko S.V., Moffat A.F.J., Eenens P.R.J., Hill G.M., Grandchamps A.: 1995, Astrophys. J., **450**, 811

Mayer P., Hadrava P., Harmanec P., Chochol D.: 1991, Bull.Astron. Inst.Czechosl., **42**, 230

Svechnikov M.A.: 1954, Graduation paper, Leningrad State Univ.

Svechnikov M.A.: 1986, A catalog of orbital elements, masses and luminosities of close binary stars, Irkutsk, University press

Swings P.: 1936, Zr. f. Ap., **12**, 40