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On one algorithm of fitting of the function, preassigned by table, to tabular defined points.

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Abstract. An algorithm for selection of parameters on the basis of the sum of squares of discrepancies (squariance) minimization is described for tabular defined functions. Possibilities of application of the algorithm for some astrophysical problems are presented.

Key words: data reduction: algorithms

1. Introduction

In applied astrophysics one often faces a problem of estimating shift, rotation, and scale parameters for a number measured quantities using a known tabular preassigned function. The problem can be formulated as one of selecting optimum configurations or considered to be a particular case of the transport problem. However, taking into account the very large number of elements (e.g. pixels in observational records) in the applied tasks a more simple and speedy approach in the selection of parameters would be favored. The most complicative aspect of the problem is the presence of noises making difficult the convergence of the method, for both the tabular preassigned function and the measured values.

To determine the parameters a variation of the least square method suggested to be used for the parameters determination, namely minimizing the sum of squares of discrepancies (squariance) between the measured values and values of the tabular preassigned function.

2. Algorithm description

With no allowance for rotation (see below) the minimization will take the simplest form:

$$R = \min_{i,j,p} \sum_k \sum_l (f_{i+\Delta_k,\, j+\Delta_l} - c_p a_{k,l})^2, \qquad (1)$$

where f is the tabular preassigned function; a is the sample of measured values; c is the scaling factor; i, j $(i = i_{min}, i_{max}; j = j_{min}, j_{max})$ are the indices of the arguments of the function f; k, l $(k = k_{min}, k_{max}; k = k_{min}, k_{max})$ are the indices of the arguments of the measured values a; p $(p = p_{min}, p_{max})$ are the indices of the table of the values of the scaling factor c; Δ_k, Δ_l are the differences of indices of the function f arguments, corresponding to relative positions of arguments of the measured function a on the axes of the function f.

The limits of measurement of the parameters are chosen so as to reduce the number of operations being performed. It can be increased with the use of the iteration method. First the calculation steps are chosen sufficiently large: 5–10 % of difference of the limits of the function arguments and values. Then the steps are decreased 3–5 times untill the variation of R (formula (1)) is within a given ϵ.

2.1. Estimation of errors of parameter determinations

As was noted by Fomalont (1989), "error estimates which are obtained directly from most fits should be viewed with skepticism". Using σ as the post-fit r.m.s. error, we, however, can estimate parameters for most models approximately (Fomalont, 1989),
$\Delta C = \sigma$, scaling factor,
$\Delta l = \sigma\theta/2C$, position,
where
$\theta = \frac{1}{F}\sum(l_i - l)^2 a(l_i)$ is the width,
$l = \frac{1}{F}\sum l_i a(l_i)$ is the mean position,
$F = \sum a(l_i)$, and l_i is the argument of a.

3. Examples

Below examples of use of this algorithm are given for two completely different astrophysical problems.

In Fig.1 an example is shown of inscribing the tabular preassigned radio telescope spread function, computed by Korzhavin (1977) method, in a real record of the source passage through the beam of RATAN-600. This function is operated in the flexible astronomical data processing system FADPS (Verkhodanov, 1993;

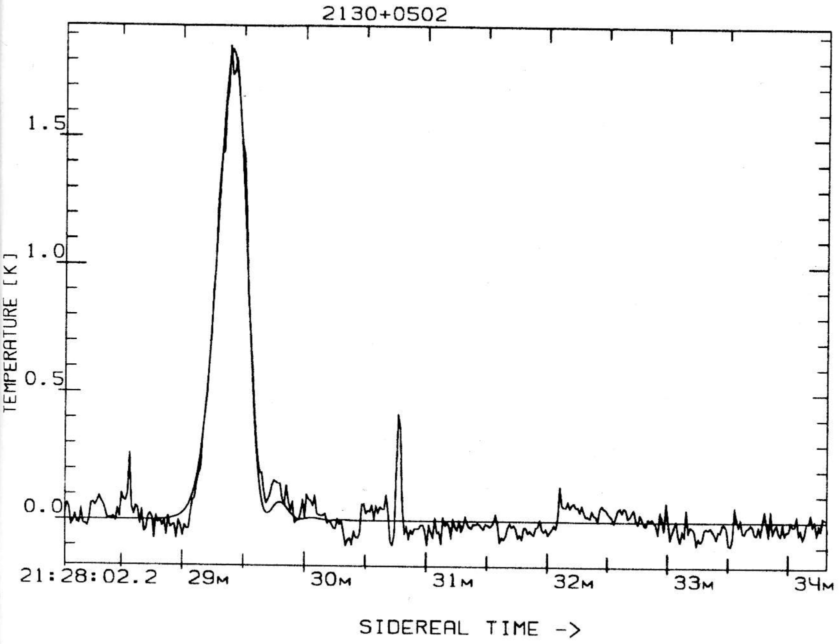

Figure 1: *Result of inscribing of the tabular preassigned function in the real record of the source passage through the RATAN-600 beam pattern. A wave length is 31 cm. There is a sidelobe on the right of the source.*

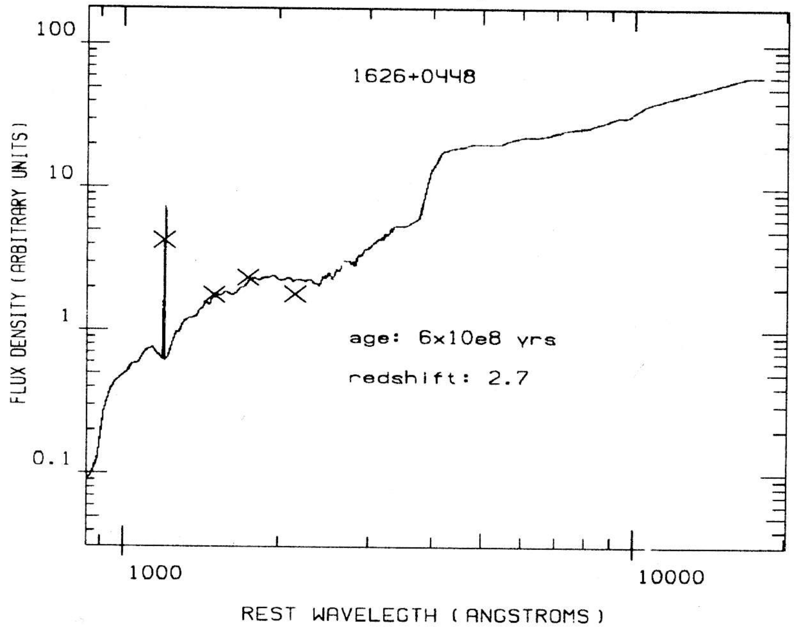

Figure 2: *Result of points position definition for magnitudes, measured photometrically in four filters BVRI, at the spectral energy distribution (model of Chambers and Charlott, 1990).*

Verkhodanov et al., 1993) at RATAN–600 (the procedure *beamap* – approximation with the antenna beam pattern). Using the method, described by formula (1), we can measure the position, amplitude and halfwidth with iterations. Taking into account that while fitting the calculated beam pattern to select the position, a step equal to the pixel size was used, to detect the position near the minimum R, the step was decreased by a factor of 3, the real records were interpolated. To estimate the most probable values of all parameters we made a parabolic approximation using 7 points near the minimum of R at each iteration step. An extremum of this parabola was used as a start point of the next iteration. The distinction of the algorithm in this case consists in that the beam pattern halfwidth, and hence its shape, changes as a parameter. That is done by the interpolation and reappropriation of pixel values. The desired accuracy of approximation (about 1–2%) is achieved after the second iteration.

The second example (Fig. 2) demonstrates the determination of redshifts from the measured photometrical magnitudes by the selection of the position of these magnitudes on the curve of the spectral energy distribution for different models of ages of radio galaxies. This task is solved in estimation of photometric redshifts (Parijskij et al., 1995, 1996).

In this example we defined the shift in the tabular preassigned curve. This task is one of the simplest for the proposed method because only one parameter is changed.

4. Additional possibilities

Another possible application of this method consists in algorithmization of the procedure of configuration selection of astrometric calibration using standard stars in the CCD frame. This can be done in the following manner. Consider, for example, 180 possible positions (via 1 degree depending on rotation angle) of a group of standard stars and project their corresponding vectors on the X axis. The predetermined positions of the frame objects are projected on the same axis. After that we pick up the projections with a minimum squariance using shift and scale variations for all the 180 projections. Near the minima that have been found the smaller steps are specified and the process is repeated untill a stable result is achieved.

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