Bull. Spec. Astrophys. Obs., 1996, 39, 66–77

Interpretation and solution of the light curve of the Wolf–Rayet eclipsing binary CQ Cep

T. A. Kartasheva[a], **M. A. Svechnikov**[b]

[a] Special Astrophysical Observatory of the Russian AS, Nizhnij Arkhyz 357147, Russia
[b] Ural State University

Received July 12, 1994; accepted December 2, 1994.

Abstract. As distinct from the authors of numerous solutions of the CQ Cep light curve, who consider it to be caused mainly by the effects of ellipsoidality and eclipse of the components, we persist in considering the light curves of this very close system ($A = 20\ R_{\odot}$) to be compound. We believe that about half of the amplitude of most of the CQ Cep light curves is due to the light variations of the extremely density–inhomogeneous common envelope of the system, and only the remaining half is associated with the effects of ellipsoidality and eclipse of the components. As a result of the light curve correction for the orbit eccentricity and introduction into consideration of the light of the third component (the light of the common envelope), a more accurate solution has been obtained for the July–August 1937 CQ Cep light curve which has the least amplitude and, probably, is less distorted by the envelope inhomogeneities. The analysis of this solution allowed us to understand the nature of the high–amplitude light curves of the system, present a model of CQ Cep, and specify spectral classification of the companion.

Key words: stars: individual:CQ Cep – photometry: Wolf–Rayet stars – light curve

A purposeful photometric study of CQ Cep was started in the middle of the century after Gaposchkin (1942) found the star light variability, and in 1944 he discovered it to be an eclipsing binary (Gaposchkin, 1944). For the next four decades a great number of light curves of CQ Cep were obtained by different authors using mainly the photoelectric methods of observation (Hiltner, 1950; Svechnikov, 1954; Chugainov, 1960; Ishchenko, 1963; Kartasheva, 1966; Guseinzade, 1967; Khaliullin, Cherepashchuk, 1970; Kartasheva, 1972; Khaliullin, 1972; Kartasheva, 1976; Kurochkin, 1979; Antokhina et al., 1982; Stickland et al., 1984; Shylaja, 1986; Kartasheva, 1987; Harvig, 1987; Stickland et al., 1988).

It is the photoelectric light curves, being the most accurate, that the attention of the researchers has so far been mainly paid to. However, in our opinion only the whole aggregate of data on the behaviour of the CQ Cep light, including "accidental" photographic observations of the first half of the century, allows one to see the true scale of the star's photometric variability, understand the nature of its low and high amplitude light curves and thus find the key to solution of these curves.

There are two series of long–term CQ Cep light photographic observations carried out at the Harvard Observatory (≈ 1800 light estimates obtained from 1901 to 1942) (Gaposchkin, 194 and at the Sternberg Astronomical Institute (SAI (> 500 light estimates obtained from 1899 1962) (Kurochkin, 1979). Unfortunately, neith Gaposchkin nor Kurochkin, who had measur the plates, published individual light estimate Gaposchkin (1944) presented the data on the me photographic light curve for forty years of observ tions (Fig.1a), and only a few data on 15 independe light curves which were used for plotting the averag Concerning the measurements of plates of the SAI stills library, Kurochkin provides in his paper onl light curves of the system, obtained at observatio in the first half of the century. However, we have t tables of his CQ Cep light estimates he generous made available.

The basic result of Kurochkin's study, in our op ion, is that he has found the light curve amplitu of the system considerably variable. This shows most vividly when comparing the light curves of Ju August 1937 (series mT, 146 light estimates) a August–September 1938 (series mT, 66 estima (Figs.1c,d). Figs.1b,e display another two light cur of CQ Cep plotted from the other series of plates the SAI's stills library: 1b — the light curve of 18 1908 (series S, 31 light estimates); 1e — the m light curve for 1899–1942 (series S and T, about

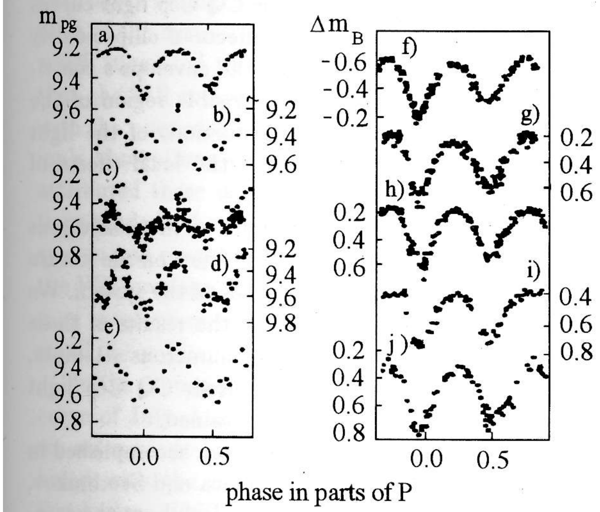

Figure 1: *The photographic light curves of CQ Cep obtained from the observations of the first half of the century (left): a) Harvard observations from 1899 to 1942 (Gaposchkin, 1944); b) – e) — SAI observations (Kurochkin, 1979): b) observations from 1899 to 1908; c) observations of July–August 1937; d) observations of August–September 1938; e) observations from 1899 to 1942. The photoelectric B light curves of CQ Cep obtained in the second half of the century (right): f) observations from 1958 to 1959 (Chugainov, 1960); g) observations from 1964 to 1965 (Guseinzade, 1967); h) observations of February–May 1969 (Kartasheva, 1972); i) summer observations of 1980 (Antokhina et al., 1982); j) observations of February–September of 1982 (Harvig, 1987)*

light estimates). The latter combines the observations obtained at the SAI at about the time of the Harvard observations. Although the accuracy of this curve is much worse (because of the small number of measurements) than that of the average light curve plotted by Gaposchkin, the depths of the same name minima are similar.

The reality of episodic diminution in the amplitude of the system light curves was confirmed later by the photoelectric observations of 1980 (Antokhina et al., 1982) that provided UBVR curves with essentially lower amplitudes of the light variations. (In particular, the depths of the minima for the B curve were $\Delta m_{minI} \approx \Delta m_{minII} \approx 0^{\rm m}.33$). This can be well seen under examination of the right part of Fig.1, where five photoelectric B light curves are presented, including the curve obtained by Antokhina et al. (Fig.1i).

Aside from the "amplitude instability" of the CQ Cep light curve from one observational season to another, the instability from cycle to cycle, which creates a rather high scattering of points in the light curves, is also characteristic of the system. This kind of instability noted in the observations of Hiltner (1950), Chugainov (1960), Kartasheva (1972), and Stickland et al. (1984) was especially pronounced in the observations of Guseinzade (1967) and our narrow–band observations in 1975 (Kartasheva, 1976; 1987), when the light variations from cycle to cycle exceeded $0^{\rm m}.1$.

Finally, short–time light flashes ($\Delta t \approx 2^h$) were observed in two series of observations. They had an amplitude of $0^{\rm m}.15$ – $0^{\rm m}.20$ in the photographic and visual regions and about $0^{\rm m}.35$ in the near ultraviolet. Their more detailed description is given in the papers by Kartasheva (1972) and Kurochkin (1979).

The CQ Cep light curve instabilities mentioned above were accompanied by changes in its shape. We speak about the variable degree of asymmetry of the two minima (both from season to season and from cycle to cycle), shifts of the secondary minimum and the two maxima from their normal positions in phase, inequalities of the light of the maxima occurring from time to time, and, at last, local absorptions on some CQ Cep light curves at the secondary minimum phases.

It is interesting that there exists a direct correlation between the light curve amplitude and the degree of asymmetry of its minima: the larger the amplitude, the greater the degree of asymmetry.

The possibility of direct superimposing of the light curves of July–August 1937 and August–September 1938 (the observations refer to one and the same series) allowed Kurochkin to find one more interesting feature: the curves differ in the height of the maxima, while their minima are nearly coincident.

And finally, close examination of the lowest–amplitude light curve of July–August 1937 showed that a rather flat maximum I ($0^{\rm P}.25$) normally located in phase is combined with gas flux absorption strictly localized in phase ($0^{\rm P}.53 - 0^{\rm P}.92$).

What caused the instability of the CQ Cep light curve?

The orbital eccentricity and the motion of the apsis line detected when inspecting the (O–C) diagram lead to periodical deviations of the secondary minimum's centre from phase $0^{\rm P}.5$ and create small asymmetry of the light curves in the minima. However, this effect can not account for the considerable instability observed in the light curve of the system. The principal reason for the instability is likely to be associated with the fact that the WR envelope in this very close pair (the distance between the centres of the stars is $20 R_\odot$) is also the common envelope of the system and, therefore, very inhomogeneous in density.

Probably, as a result of stellar wind collisions, a hot gas condensation is formed between the components, which produces essential additional radiation in the visible region of the continuum. It is in this state that the system shows asymmetric, high–amplitude light curves which are obviously compound. About half of the amplitude of the light curve is due to the light variations of this gas condensation undergoing eclipse, while only half of the amplitude is associated with the effects of ellipsoidality and eclipse of the components.

The appearance of the low–amplitude light curves of the system ($\Delta m < 0^{\rm m}.4$), suggesting that the source of additional radiation in the continuum weakened or completely disappeared, is, evidently, caused either by the reduced activity of the pair's components or by the temporary simplification of the envelope structure as a result of gas ejection from the system. The latter is confirmed by both photometric (Kartasheva, 1972; Kurochkin, 1979) and spectroscopic (Hiltner, 1944) investigations. The analysis of the (O–C) diagram of CQ Cep also points to the considerable gas ejection from the system (Kartasheva and Svechnikov, 1991). Such is the supposed model of the processes developing in the system. On the basis of this model we regard all high–amplitude light curves of CQ Cep ($\Delta m \geq 0^{\rm m}.4$) as the most distorted and unfit for the photometric solution.

The low amplitude of the CQ Cep light curves in the infrared region of the continuum (J and K curves, Stickland et al., 1984), where the radiation from the hot gas condensation must be small or absent, is also consistent with this model. Indeed, the UBV curves in the observations of Stickland et al. (1984) show high–amplitude light variations ($\Delta m \geq 0^{\rm m}.4$), according to Table 6 and Fig.15 of the paper cited, while the amplitude of the J curve ($\lambda_{eff} = 12400\mathring{A}, \Delta m_{\rm minI} \approx \Delta m_{\rm minII} = 0^{\rm m}.33$) has a certain mean value, and the K curve ($\lambda_{eff} = 22200\mathring{A}, \Delta m_{\rm minI} = 0^{\rm m}.245, \Delta m_{\rm minII} = 0^{\rm m}.23$) is closer in amplitude to the photographic light curve of the system of July–August 1937. To our regret Stickland and his colleagues did not publish the Table of CQ Cep light estimations made in September 1981 in the infrared region of the continuum.

At the present time there is a series of solutions of the CQ Cep light curve (Gaposchkin, 1944; Kartasheva, 1974; Kartasheva and Svechnikov, 1974; Lipunova and Cherepashchuk 1982; Long et al., 1983; Stickland et al., 1984; Kartasheva and Svechnikov, 1988; Antokhina and Cherepashchuk, 1988; Harvig, 1989), including solutions accomplished by the most up–to–date methods, which take into account the extended WR–atmosphere.

If we disregard the solution of Gaposchkin, which is of purely historical interest, as well as our two solutions, the remaining ones unite two features: first, all of them are obtained for high–amplitude and asymmetric photoelectric curves (or a somewhat mean curve, which hardly alters the situation); and second, they are united by the common approach of the researchers to the interpretation of the light curves. It is considered that high–amplitude CQ Cep light curv are mainly determined by the effects of ellipsoidali and eclipse of the components, the envelope's contr bution to the radiation in the visible region of th continuum is small, and all distortions of the ligh curves are associated only with the local effects o gas fluxes.

We do not discuss the details of these solution since we have a radically different view on the natur of the high–amplitude light curves of the system. W note only that the dispersion of the results of these solutions is high, and despite the numerous attempts no confident, universally recognized, CQ Cep ligh curve solution has so far been obtained.

Our CQ Cep light curve solution accomplished i 1972 (Kartasheva, 1974; Kartasheva and Svechnikov 1974) on the basis of the correct idea about the compound character of all the light curves of the system known at that time was methodically imperfect The light curve was broken down into "stellar" and "envelope" using a high coefficient of ellipsoidality o the components, following from the assumption tha the system is contact. The graphical determination o the ellipsoidality coefficient, that became possible after the CQ Cep light curve of July–August 1937 wa published by Kurochkin in 1979, showed that it wa overestimated by about a factor of 2. It turned ou impossible to repeat the process of decomposition o the high–amplitude light curves of the system wit this refined coefficient, since the bend points on th curves rectified with an essentially lower ellipsoidalit coefficient were not defined clearly.

On the basis of the analysis of the (O–C) diagra of CQ Cep and the results of the paper by Kurochki (1979) we suggested that the lowest–amplitude photographic light curve of the system of July–Augu 1937 is the least distorted by radiation of the circum slellar gas.

For a tentative solution of this curve (Kartashev and Svechnikov, 1988) we took only a part of (phases $0^{\rm p}.92 - 0^{\rm p}.0 - 0^{\rm p}.53$) which was not distorted b the circumstellar gas. Each of the minima was considered separately. For the first time it was assume that not only the WR component but also the O component has a complex structure.

At first glance the calculations made gave a re sonable solution. Complying with the requirement the O star filling its inner Roche lobe (IRL), it show a good agreement of the estimates of the orbit inc nation, following from the separate solutions of t minima, and a good theoretical representation of t observed light curve. However, under close consider tion we were not satisfied with the solution obtaine It seemed unnatural that the nucleus of the WR st had filled a rather small area of the IRL, and th the O star's luminous photosphere had considerab overfilled its outer critical surface. The most imp tant shortcoming of the solution was the low light ratio of the system's components ($L_{WR}/L_O = 0.49$), which suggested that the companion is $0^{\mathrm{m}}\!.8$ brighter than the WR component. This in no way coordinated with the poor visibility and even the lack of companion's absorptions in the spectra of the system. We attributed these discrepancies to the two simplifications made in the solution of 1986: 1) the orbit of the system was considered circular ($e = 0$); 2) the light of the WR envelope, which at the same time is the common envelope of the system, was assumed negligibly small ($L_3 = 0$).

In the more accurate solution of the CQ Cep light curve of July–August 1937, presented in this paper, we attempted: 1) the small orbit eccentricity combined with the apsis line motion (Kartasheva and Svechnikov, 1989); 2) to consider the possibility of an appreciable contribution of the envelope to the radiation of the system in the visible region of the continuum.

The phases of the light curve (θ) were computed first by the linear formula:

$$T_{min I} = 2428747^{\mathrm{d}}\!.474 + 1^{\mathrm{d}}\!.64126224E,$$

which we deduced employing the results of our paper (Kartasheva and Svechnikov, 1991) and which assumed uniform motion of the star on the orbit.

Further, utilizing the fact that with the small eccentricity value the moments of minima in the elliptical orbit are almost coincident with the moments of conjunction ($\nu \approx 0°$ at the moment of the main minimum and $\nu \approx 180°$ at the moment of the secondary minimum), we found a simple relation between the angular phase (θ) and the true longitude in the elliptical orbit (ν):

$$\theta = \nu - 2e\cos(\nu - \omega) + h(1 + csc^2 i)$$

(Zverev et al., 1947). For the CQ Cep's orbit, which shows alongside with the eccentricity the apsis line motion ($e = 0.02, \dot{\omega} = 0^{\circ}\!.03$ for the period, $U = 54$ years) (Kartasheva and Svechnikov, 1989), the formula of connection for the middle of 1937 is as follows:

$$\theta = \nu - 0.04\cos(\nu - 177^{\circ}\!.5) - 0.044.$$

With the help of the last formula we made the transition from the uniform motion to the true non-uniform motion in the elliptical orbit ($\theta \to \nu$), which, having restored the consistency between the light and the angular phase, put the secondary maximum in its place and eliminated the asymmetries of the minima associated with the orbit ellipticity.

All the data on the July–August 1937 average light curve of the system are listed in Table 1.

We made a series of assumptions concerning the light of the third component (L_3, the light of the common envelope that does not take part in eclipse). It was set equal to 0.0, 0.1, 0.2, 0.3, 0.35, 0.4 at

$$L_{WR\,photosph} + L_O + L_3 = 1.$$

In order to reduce the curve, burdened with the light of the third component, to the form appropriate for solution, we subtracted from all the light values L_3, and then by multiplying the obtained differences by the coefficient $(1 + L_3/(L_{WR\,photosph} + L_O)$ reduced all the light values to a new maximum value

$$l_{max} = [1 + \tfrac{L_3}{L_{WR\,photosph} + L_O}] \cdot (L_{WR\,photosph} + L_O) =$$

$$= L'_{WR\,photosph} + L'_O = 1.$$

For each value of L_3 (after the above light curve transformation) the rectification coefficients were defined graphically from the shape of its first maximum. For the sake of simplicity, because the photographic curve was being solved, the "U"–hypothesis was considered: uniform distribution of brightness over the discs of the stars was assumed.

The light curve was only corrected for the effect of ellipsoidality of the components. The reflection effect was disregarded since it was small in very close systems whose components differed slightly in luminosity.

The allowance for the gravitation effect at $x = 0$ ("U"–hypothesis) and $y = 1$ (hot stars) gave $N = 2$ and led to the following relation between the photometric ellipsoidality coefficient (z_{phot}), the geometric ellipsoidality coefficient (z_{geom}) and the eccentricity of the meridian cross–section (ε) : $z_{geom} = 1/2 z_{phot} = \varepsilon^2 \sin^2 i$, used in the subsequent solution.

The solution itself of the corrected light curve was performed by the the Merrill–Russel method for the circular orbit (because e is small) in the scheme of two similar ellipsoids. Each of the minima was considered separately. As in the tentative solution of 1986, both components were assumed to have a complex structure. In consequence of this fact the stars were considered to have different sizes, being eclipsing and eclipsed objects. Each of the minima in this case would give its pair of photometric elements (k_1, α_{01} and k_2, α_{02}) which would acquire definite sense and be connected by a definite relation of depths under different assumptions of types of eclipses.

Since the O companion is sure to fill its IRL, while the dark nucleus of the WR star is small, then in the secondary minimum (WR star is in front) the only type of eclipse is real, $S \to L$ (a small star eclipses a large star), and the meaning of k_2 is unambiguous ($k_2 = a_{WR\,dark}/a_{O\,light}$).

In the main minimum (O star is in front) both types of eclipse are possible. In the case of eclipse of

Table 1:

No.	m	ℓ ($m_{max} = 9^{\rm m}\!.46$)	Phase (in portions of P)	θ°	ν°
1	$9^{\rm m}\!.66$	0.833	$0^{\rm p}\!.991$	356.76	356.86
2	9.64	0.849	0.035	12.60	12.80
3	9.61	0.871	0.060	21.60	21.93
4	9.57	0.908	0.103	37.08	37.64
5	9.53	0.938	0.137	49.32	50.34
6	9.48	0.986	0.184	66.24	67.88
7	9.47	0.991	0.256	92.16	94.86
8	9.46	0.998	0.296	106.53	109.86
9	9.48	0.986	0.319	114.84	118.42
10	9.49	0.973	0.352	126.72	130.69
11	9.52	0.949	0.374	134.64	138.83
12	9.59	0.891	0.446	160.56	165.24
13	9.60	0.876	0.481	173.16	177.85
14	9.60	0.876	0.516	185.76	190.19
15	9.60	0.879	0.555	199.80	204.24
16	9.57	0.901	0.653	235.08	240.59
17	9.50	0.961	0.692	249.12	252.14
18	9.48	0.980	0.712	256.32	259.06
19	9.48	0.986	0.742	267.12	269.46
20	9.47	0.991	0.781	281.16	282.94
21	9.56	0.916	0.851	306.36	307.30
22	9.58	0.894	0.916	329.76	330.13
23	9.63	0.855	0.951	342.36	342.56
24	9.64	0.846	0.972	349.92	350.06

type $L \rightarrow S$ $k_1 = a_{WR\,light}/a_{O\,dark}$ and the depth of the minima will be related as follows:

$$\frac{1-\lambda_1}{\alpha_{01}} + \frac{1-\lambda_2}{k_2^2\alpha_{02}} = 1.$$

If in the main minimum eclipse of type $S \rightarrow L$ occurs, then $k_1 = a_{O\,dark}/a_{WR\,light}$ and

$$\frac{1-\lambda_1}{k_1^2\alpha_{01}} + \frac{1-\lambda_2}{k_2^2\alpha_{02}} = 1.$$

As a rule, $(1-\lambda_1)$ and $(1-\lambda_2)$ are the depths of the main and secondary minima, α_{01} and α_{02} are the maximum photometric phases of the eclipses for the main and secondary minima, and k_1 and k_2 are the ratio of the major semi–axes of the components for the main and secondary minima. Following Kron and Gordon (1950), a_{dark}, b_{dark} imply the sizes of effectively non–transparent discs (nuclei) of the stars, where $\tau > 1$; a_{light}, b_{light}, the sizes of dense parts of the electron scattering photospheres emitting in the continuum.

The light curve solution was started with an independent consideration of the main minimum which was better represented by observations. A more general case was considered, the one of partial eclipse. A series of pairs k_1, α_{01} was determined from the light curve shape and a suitable series of possible solutions was found: component sizes ($a_{O\,dark}, b_{O\,dark}; a_{WR\,light}, b_{WR\,light}$), orbit inclination ($i_1$) and light of the WR component ($L'_{WR\,photosph}$) at different assumptions as to the eclipse type. The proper solution was chosen by proceeding from the requirement of filling with the O star nucleus its IRL. At the same time the type of eclipse was chosen automatically, and the values of i and $L'_{WR\,photosph}$ were fixed. Besides this, the solution had to give a good theoretical representation of the light curve in the main minimum.

The secondary minimum, which was worse represented by observations, was considered separately but the solution was already dependent on that for the main minimum. Here the choice of solution was made proceeding from the following requirements:

- the value of the O component light (L'_O) must be close to $(1 - L'_{WR\,photosph})$;
- the orbit inclination estimate (i_2) must be close to the estimate obtained from the solution of the main minimum (i_1);
- the solution must ensure good theoretical representation of the secondary minimum.

The whole aggregate of solutions for the main a

Table 2:

L_3	0.0	0.1	0.2	0.3	0.35	0.4	0.27
z_{phot}	0.16	0.17	0.20	0.22	0.23	0.25	0.22
	Main minimum (partial eclipse of $S \rightarrow L$ type)						
k_1, α_{01}	0.72 0.53	0.75 0.48	0.75 0.47	0.75 0.45	0.75 0.45	0.77 0.42	0.75 0.46
$a_{WR\,light}$	0.572	0.563	0.564	0.566	0.565	0.559	0.561
$(b_{WR\,light}$	(0.542)	(0.530)	(0.525)	(0.522)	(0.519)	(0.509)	(0.520)
$a_{O\,dark}$	0.412	0.422	0.423	0.424	0.424	0.431	0.424
$(b_{O\,dark})$	(0.390)	(0.397)	(0.393)	(0.391)	(0.389)	(0.392)	(0.391)
	$(IRL_{O-st} = 0.391)$						
$i^\circ{}_1$	61.2	60.3	59.9	59.1	59.3	58.6	59.1
$L'_{WR\,photosph}$	0.33	0.39	0.44	0.54	0.61	0.67	0.51
$L_{WR\,photosph}$		(0.35)	(0.35)	(0.38)	(0.40)	(0.40)	(0.375)
	Secondary minimum (partial eclipse of $S \rightarrow L$ type)						
k_2, α_{02}	0.28 0.84	0.34 0.75	0.38 0.68	0.54 0.51	0.62 0.52	0.75 0.45	0.48 0.59
$a_{O\,light}$	0.641	0.625	0.616	0.565	0.544	0.510	0.584
$(b_{O\,light})$	(0.605)	(0.589)	(0.573)	(0.522)	(0.503)	(0.468)	(0.541)
$a_{WR\,dark}$	0.179	0.213	0.234	0.305	0.337	0.383	0.283
$(b_{WR\,dark})$	(0.169)	(0.200)	(0.218)	(0.282)	(0.311)	(0.356)	(0.262)
	$IRL_{WR-st} = 0.357$						
$i^\circ{}_2$	59.3	59.6	59.5	59.9	62.1	62.7	59.8
L'_O	0.67	0.61	0.56	0.46	0.39	0.33	0.49
(L_O)		(0.55)	(0.45)	(0.32)	(0.25)	(0.20)	(0.355)
L_{WR}/L_O	0.49	0.82	1.22	2.13	3.00	4.00	1.81
$J_{WR\,photosph}/J_O$	0.62	0.79	0.94	1.17	1.45	1.69	1.08
$i^\circ{}_{mean}$	59.8	60.0	59.7	59.5	60.7	60.7	59.5
$\Delta i^\circ = i_2 - i_1$	-1.4	-0.3	-0.4	+0.8	+2.8	+4.1	+0.7

secondary minima of the system light curve of July–August 1937 under different assumptions concerning the light of the common envelope (L_3) is presented in Table 2. Here are listed the values of photometric ellipsoidality coefficients of the stars (z_{phot}). Next are given (for each of the minima): ratio of major semi-axes of the components (k) and maximum photometric phase of eclipse (α_0), sizes of both semi-axes of the components (a, b), orbit inclination values (i), and light of the eclipsed star (L' and L). The last lines of Table 2 present the ratio of the total light of the WR component ($L_{WR\,photosph} + L_3$) to the O star light, the surface brightness ratio of the components

$$\left(\frac{J_{WR\,photosph}}{J_O} = \frac{L'_{WR\,photosph}\, a^2_{O\,light}}{L'_O\, a^2_{WR\,light}} \right),$$

the mean value of the orbit inclination, $i_{mean} = \frac{i_1 + i_2}{2}$, and the difference of orbit inclination values following from the solutions of the main and secondary minima ($\Delta i = i_2 - i_1$).

Part of the results are displayed in Fig.2. Fig.2a presents the variation of component sizes (their minor semi-axes "b") versus L_3. The straight lines indicate the sizes of the inner and outer Roche lobes of the components (y–coordinates). The mass ratio needed to enter the Tables of Roche models (Plavec and Kratochvil, 1964) was taken from the paper by Kartasheva and Snezhko (1985b). The variation of the difference $\Delta i = i_2 - i_1$ with L_3 and the variation of the light ratio of the components $L_{WR}/L_O = (L_{WR\,photosph} + L_3)/L_O$ with L_3 are shown in the Fig. 2b and Fig.2c, respectively. When examining Fig.2b, one would formally prefer the solution with $L_3 = 0.2$, when the difference in orbit inclinations, obtained from the solution of the main and secondary minima, reaches minimum ($\Delta i = 0^\circ$). However, taking into account the error of defining i ($\delta i = \pm 1\overset{\circ}{.}5$), this choice seems to us ungrounded. We found the choice of L_3 and the light curve solution corresponding to it from the dependence of the light ratio of the components on L_3 more reliable. The knowledge of the light ratio of the components required for this induced us to carry out a spectrophotometric investigation of lines in the system's spectra taken in 1981 and 1987 at the phases close to the first maximum of light. The results of the investigation (Kartasheva, 1996) allowed us to

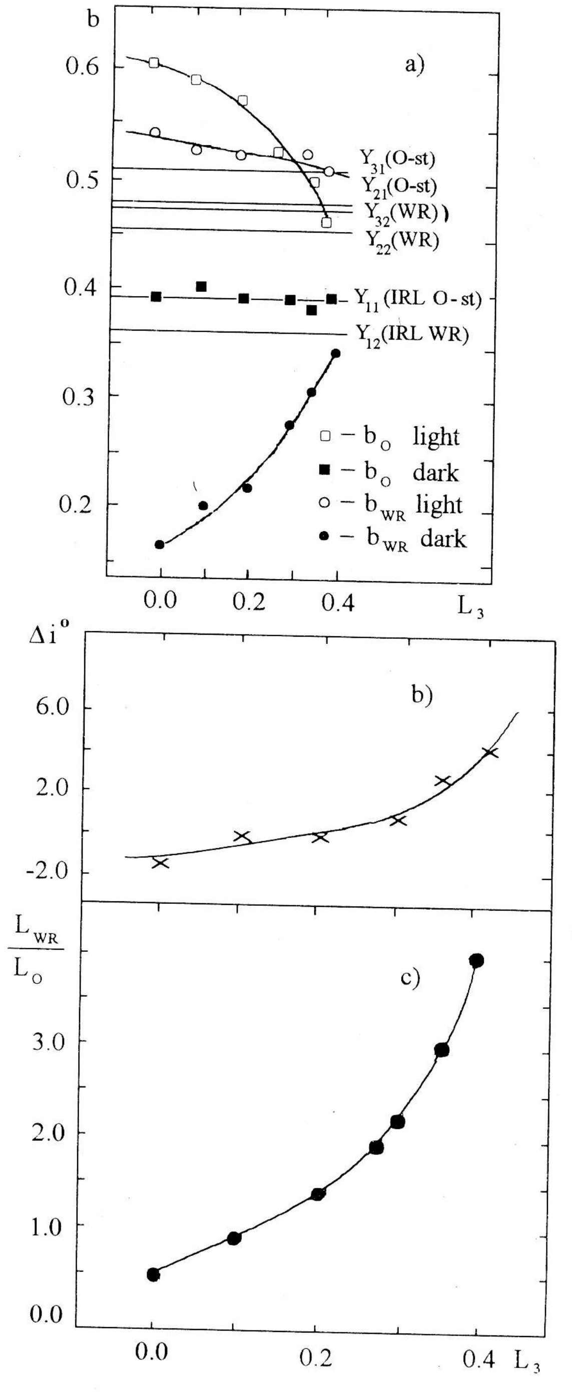

Figure 2: *The size variation of the components (their minor semi-axes) with* L_3. *The straight lines indicate the sizes of the inner and outer critical Roche lobes along the y-coordinate (a). The variation of the difference* $\Delta i = i_2 - i_1$ *with* L_3 *(b), the variation of the light ratio of the components* L_{WR}/L_O *with* L_3, $(L_{WR} = L_{WR\,photosph} + L_{WR\,env})$ *(c).*

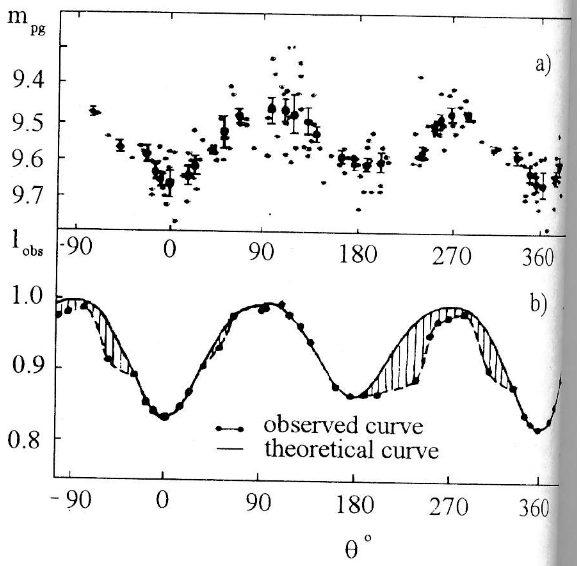

Figure 3: *The observed light curve of CQ Cep July–August 1937 (large circles indicate the me points against the background of individual light timates) (a). The same light curve in the coordinat* $l_{obs} - \theta$. ***The solid line shows the theoretical light cur following from the obtained photometric solution*** $L_3 = 0.27$ *(b).*

conclude that the WR star light was about two ti that of the companion (L_{WR}/L_O = 1.81) which likely to belong to faint giants. This result and Fig helped us to find a correct value of the WR envelo light, $L_3 = 0.27$. The solution of the 1937 light cur corresponding to this value is presented in the l column of Table 2.

The discrepancy of the orbit inclination val that follow from the final solution of the main a secondary minima is +0°.7, which lies within the rors of i determination. It can be seen from Fig that with an error of determining the relative co ponent sizes of ±0.005 the WR component's nucl does not essentially fill its IRL, and the size of electron scattering photosphere considerably exce the size of its outer critical Roche surface. On contrary, the O star's photosphere fairly well fits outer critical Roche surface.

The theoretical representation of the CQ Cep li curve of July–August 1937 is given in Fig.3b. Ap from the light curve region suffering strong distort by the gas flux (phases $0^P.53 - 0^P.92$) one more anom seems to have been revealed: weaker absorption phases $0^P.08 - 0^P.19$, which is likely to arise when less dense part of the gas flux is projected onto WR star.

Fig.4 depicts the system model constructed fr the results of the specified CQ Cep light curve

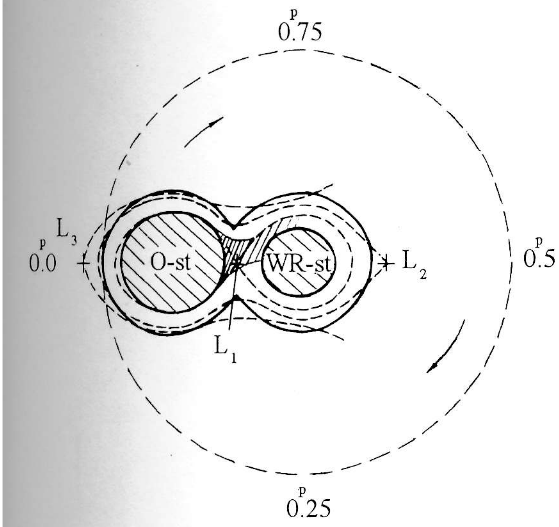

Figure 4: *The CQ Cep model following from the refined light curve solution of July–August 1937 and from the analysis of its distortion by the envelope inhomogeneities. The dashed lines present the size of the inner and outer critical Roche lobes (short dashes) and the approximate real size of the WR envelope ($R = 32R_\odot$), at which the O companion turns out completely immersed in it (long dashes).*

lution of July–August 1937 and the analysis of its distortion by the gas flux. To be more descriptive, the realistic size of the WR envelope ($R = 32R_\odot$), at which the O component is completely submerged in it, is shown in Fig.4 with long dashes.

The results of the papers by Morgan et al. (1953), Munch (1957), Roberts (1958), and Stickland et al. (1984) prove that CQ Cep belongs to the association Cep OB1. This allows one to evaluate the absolute visual magnitude of the system ($M_v = -6^{\rm m}_{.}3 \pm 0^{\rm m}_{.}4$), (Stickland et al., 1984).

The light ratio of the system's components obtained from spectrophotometric investigation of lines in the spectra of CQ Cep (Kartasheva, 1996) $(L_{WR\,photosph} + L_{WR\,env})/L_O = 1.81$ leads to a difference in magnitudes of the components $\Delta m_{phot} = 0^{\rm m}_{.}65$ and, taking into account the differences in colour indices of the stars, to $\Delta m_{vis} = 0^{\rm m}_{.}71$. This, in turn, provides the following estimates for the absolute visual magnitudes of the components:

$$M_{V\,WR} = -5^{\rm m}_{.}84$$

and

$$M_{V\,O} = -5^{\rm m}_{.}14.$$

The last estimate is consistent with our spectral classification of the companion (Sp O9 – BO) obtained from the spectrophotometry of lines (Kartasheva and Snezhko, 1985a,b) and, according to the refined calibration ($Sp - M_V$) of Straizys and Kuriliene (1981), it corresponds to the spectral class O8.5IV or O9.5III. We have chosen the latter because the mass of the CQ Cep companion ($22.05\mathcal{M}_\odot$) is closer to the mass of stars of the spectral class 09.5III than 08.5IV. (According to the relation Sp — $\mathcal{M}$ of Straizys and Kuriliene (1981) for 09.5III $\mathcal{M} = 28\mathcal{M}_\odot$, for 08.5IV $\mathcal{M} = 32\mathcal{M}_\odot$). That the companion belongs to faint giants is confirmed by the results of Kartasheva (1996), which has already been mentioned.

By employing the quantity $BC = -3^{\rm m}_{.}05$ as the bolometric correction for a 09.5III star (Straizys and Kuriliene, 1981), we defined the bolometric luminosity of the O component and then, knowing the size of the O star, we found its effective temperature ($T_{eff} = 32700K$). According to the refined calibration $Sp - T_{eff}$ of the same authors this temperature corresponds to a 09.5V-III star and confirms the above classification of the companion.

Since the bolometric corrections for WR stars show great dispersion, we found the bolometric luminosity of the WR component from the mass–luminosity relation deduced for WR stars in the paper by Maeder and Meynet (1987) using our $\mathcal{M}_{WR} sin^3 i$ estimate (Kartasheva and Snezhko, 1985b) and the value of the system orbit inclination obtained in the present paper.

The knowledge of the bolometric luminosity allowed us to determine the absolute bolometric magnitude and the bolometric correction for the WN7 component of CQ Cep ($BC_{WR} = -3^{\rm m}_{.}69$), which turned out close to the mean BC for WR stars ($\overline{BC}_{WR}) = 4^{\rm m}_{.}2 \pm 1^{\rm m}_{.}2$, van der Hucht et al., 1988).

The fact that the WR star's envelope gives an essential contribution to the system's light in the visible region of the continuum ($L_3 = 0.27$) complicated the uncertain effective temperature determination of the WR star. It is only in an indirect way, through the ratio of surface brightnesses of the stars using the formula

$$\frac{J_{WR\,photosph}}{J_O} = \frac{L'_{WR\,photosph}\, a^2_{O\,light}}{L'_O\, a^2_{WR\,light}} = \frac{e^{\frac{c_2}{\lambda T_O}} - 1}{e^{\frac{c_2}{\lambda T_{WR}}} - 1},$$

following from the assumption that the stars emit as absolutely black bodies that we could determine the effective temperature of the WR star at the level of denser layers of the extended photosphere, which emit in the continuous spectrum and are perceptibly eclipsed.

All the main physical characteristics obtained for the system on the whole as well as for the components are collected in Table 3. The location of the CQ Cep components on the Hertzsprung–Russel diagram is shown in Fig.5. The WN7 component of the system falls within the region occupied by late WN

Table 3:

	WN7	O9.5 III
i^o	59.5	59.5
$A(R_\odot)$	20.08	20.08
$a_{light}(R_\odot)$	11.3	11.7
$a_{dark}(R_\odot)$	5.7	8.5
$\mathcal{M}(\mathcal{M}_\odot)$	18.3	22.05
$(log\mathcal{M}/\mathcal{M}_\odot)$	(1.26)	(1.34)
M_v	$-5^m_.84$	$-5^m_.14$
M_{bol}	$-9^m_.53$	$-8^m_.19$
BC	$-3^m_.69$	$-3^m_.05$
$L(L_\odot)$	$5.1\cdot10^5$	$1.3\cdot10^5$
$(logL/L_\odot)$	(5.71)	(5.12)
$T_{eff}(K)$	34300	32700
$(logT_{eff})$	(4.54)	(4.51)

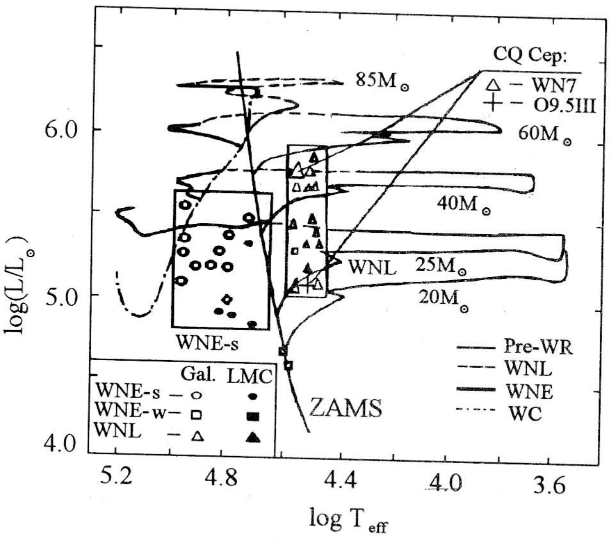

Figure 5: *The location of the CQ Cep components on the Hertzsprung–Russel diagram. The positions of different type of WR stars of the Galaxy (Schmutz et al., 1989) — open symbols, WR stars of the LMC (Koesterke et al., 1991) — filled symbols. The evolutionary tracks calculated by Maeder (1990) for solar metallicity are plotted on the diagram. The evolutionary stages are shown by lines of different types.*

stars (WNL) a little more to the left of the single star WR 78 (HD 151932, WN7). Besides this, it is very close to the evolutionary track computed by Maeder (1990) for a star of the initial mass $\mathcal{M} = 40\mathcal{M}_\odot$, which suggests that in the process of evolution the WN7 star lost about 50% of its initial mass.

As to the companion, it evolved appreciably from the zero age main sequence and lies a little lower than the track calculated for a star with the initial mass $25\mathcal{M}_\odot$. Taking into account the mass determination error ($\pm2.5\mathcal{M}_\odot$), one can say that the O component's mass did not change in the process of evolution. Using the grid of models calculated by Maeder (1990) for the star's initial mass of $25\mathcal{M}_\odot$ and initial chemical composition close to solar ($X = 0.7, Y = 0.28, Z = 0.02$) we attempted next to evaluate the age of the O component on the basis of its mass, luminosity and temperature estimates. It turned out to be $4.7\cdot10^6$ years which is in excellent agreement with the age of the association Cep OBI, of which CQ Cep is a member (According to the paper by Markarian (1953) the age of the association Cep OBI is estimated to be $4.5\cdot10^6$ years).

For a better understanding of the processes developing in the system we have considered next the CQ Cep light curve constructed by Kurochkin from the observations of August–September 1938, which are the continuation of the 1937 observations.

After the observations were consolidated in normal points, the 1938 light curve phases were corrected for orbit eccentricity and the apsis line motion, as was done for the 1937 light curve, while the magnitudes were transformed into lights, taking $m_{max} = 9^m_.46$ i.e. the same magnitude as for the light curve of 1937. In Table 4 are listed the data on the average light curve of the system for August–September 1938. In Fig.6a the light curves of July–August 1937 and August–September 1938 are presented together with the theoretical light curve which follows from the solution of the 1937 light curve. In Fig.6b light differences between the smoothed observed curves and the theoretical light curve are shown. Examination of Fig.6b allowed us to reveal the following features of the distortions of the light curves.

For the observations of July–August 1937 are noted:

1. Weak absorption is detected at phases 30°-70°, which is likely to arise when the less dense par

Table 4:

No.	m	ℓ ($m_{max} = 9^m.46$)	Phase (in portions of P)	θ°	ν°
1	$9^m.66$	0.834	$0^P.012$	$4^\circ.32$	$4^\circ.68$
2	9.62	0.867	0.066	23.76	24.26
3	9.49	0.974	0.144	51.84	52.98
4	9.37	1.080	0.213	76.68	78.71
5	9.31	1.132	0.261	93.96	96.70
6	9.31	1.132	0.315	113.40	116.91
7	9.60	0.879	0.510	183.60	188.53
8	9.57	0.903	0.592	213.12	217.67
9	9.57	0.916	0.631	227.16	231.36
10	9.62	0.861	0.660	237.60	241.48
11	9.49	0.973	0.695	250.20	253.64
12	9.39	1.064	0.737	265.32	268.19
13	9.23	1.192	0.801	288.36	290.36
14	9.38	1.073	0.845	304.20	305.63
15	9.52	0.927	0.875	315.00	316.10
16	9.59	0.883	0.918	330.48	331.21

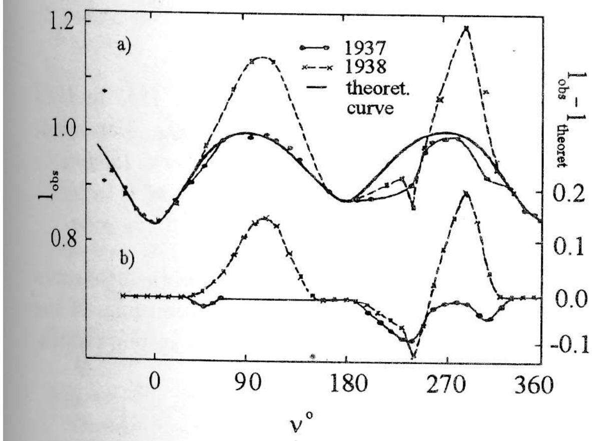

Figure 6: *The observed light curves of CQ Cep (July–August 1937 and August–September 1938) together with the theoretical curve following from the 1937 light curve solution (a). The light differences of the smoothed observed light curves and the theoretical one (b).*

of the gas flux is projected onto the WR star.

2. Absorption is noted at phases 180° – 330°, which consists of two components with maxima at phases 240° (projection of the gas flux onto the O star) and 310° (projection of the gas flux onto the WR star) with weak absorption between them (phases 270° – 290°).

For the observations of August–September 1938 are noted:

1. Additional emission is noted at phases 30° – 160° with a maximum at phase 110°.
2. Strong absorption is present at phases 180° – 255° with a maximum at phase 240°, which passes into additional emission at phases 255° – 320° with a maximum at $\nu = 290^\circ$.

A comparison of the distortions revealed on the CQ Cep light curves of 1937 and 1938 suggests that the curves are interrelated and reflect consecutive stages in the evolution of the system's common envelope. It may be supposed that for the year dividing the two observing runs the gas condensation between the components of the system considerably developed: in the middle of 1937 it showed up as an eclipsing gas flux, while in the middle of 1938 the condensation started to essentially contribute to the radiation in the continuum, which is, probably, indicative of its increasing temperature, density and extent.

In order to trace the further evolution of the gas condensation we inspected another six CQ Cep light curves obtained by different authors after 1938 in the blue region of the continuous spectrum. These curves were roughly supposed to coincide with the light curve of July–August 1937 in the centre of the main minimum. In other words, it was assumed that the source of additional light was totally eclipsed in the centre of the main minimum. Fig.7 shows the differences between the smoothed observed CQ Cep light curves and the theoretical light curve of 1937. Examining Fig.7, one can conclude that starting with the light curve of 1953 and in all the curves that follow (excluding the light curve of 1980), the additional emission of the gas condensation completely suppressed the absorption and entirely (i.e. in all the phases) "flooded" the contemporary light curves.

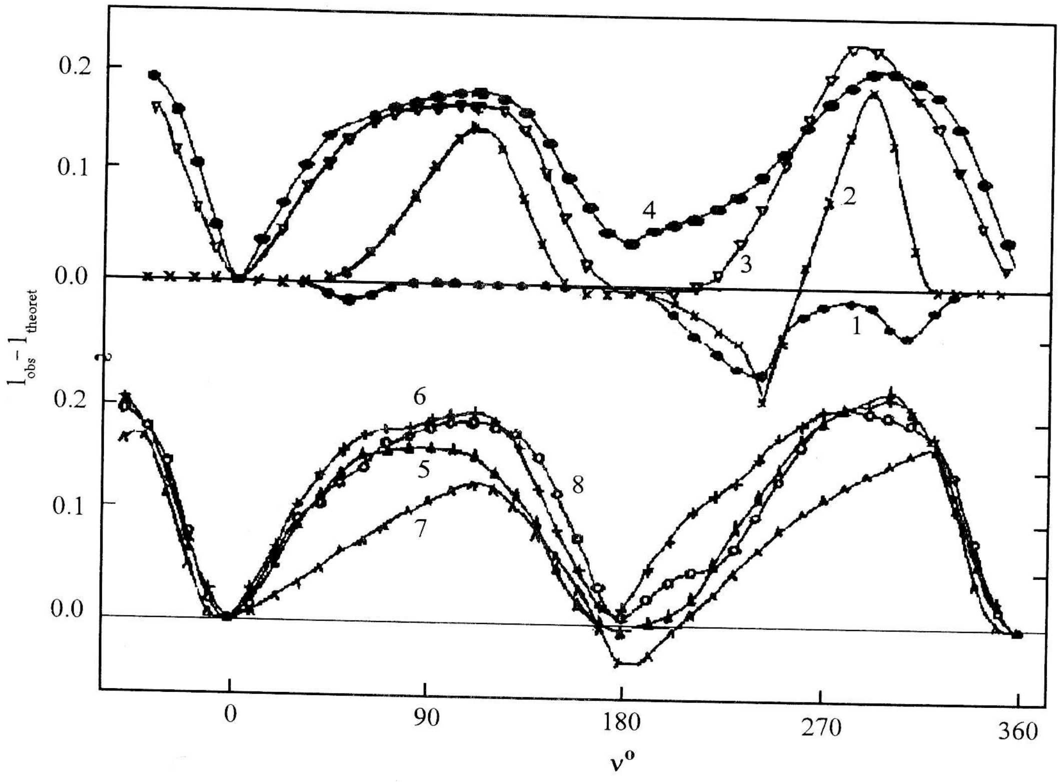

Figure 7: *The differences of the smoothed observed B light curves of CQ Cep obtained from 1953 to 19 and the theoretical curve following from the 1937 light curve solution. 1 — Kurochkin, 1937, 2 — Kurochk 1938, 3 — Svechnikov, 1953, 4 — Chugainov, 1958-59, 5 — Guseinzade, 1965, 6 — Kartasheva, 1969, 7 Antokhina, 1980, 8 — Harvig, 1982.*

Thus, it is as if a third stage in the evolution of the common envelope of the system is noted (after the stages of 1937 and 1938), which is probably characterized by subsequent growth and heating of the gas condensation between the stars.

The refined solution of the 1937 light curve and the qualitative analysis of other B light curves of the system, which is based on this solution, confirmed our assumptions made at the beginning of the paper.

Indeed, the WN7 star's envelope, being also the common envelope of the system, seems to consist of two components:

1. A rather homogeneous envelope which probably has a low density (which is justified by the numerous absorptions of the WR–nucleus found in the spectrum of the system) and therefore does not manifest itself in eclipses, but does essentially contribute to the radiation in the continuum of the system because of its large extension and
2. A quasistationary gas condensation between the pair's stars arising as a result of interaction of stellar wind of the system's components.

The variable degree of activity of the stars, apparently, determines the state and size of this gas condensation and is responsible for both the instabilities we noted in the shape and amplitude of the light cur and the variable visibility of absorption lines of t O companion in the spectrum of the system (Kar sheva, 1996).

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