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HYDROGEN MAGNETOMETER OF THE 6 M TELESCOPE. 2. SOME SYSTEMATIC ERRORS AND MAGNETIC SENSITIVITY COEFFICIENT

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ABSTRACT. *Some systematic errors, appearing during observations with the hydrogen line magnetometer of the 6 m telescope are considered in this paper. The methods of allowance for these errors and recommendations on how one can reduce them are presented. Methods of calculations and magnetic sensitivity coefficient measurement with the minimum observing time expenditure on its determination are suggested. The method of selection of exit slit optimal sizes and their location on spectral line wings is also suggested.*

INTRODUCTION

At observations on detection of stellar magnetic fields the accuracy of measurements, as a rule, does not exceed 5 $\sigma_{\bar{B}}$. Therefore, any systematic errors, not higher than 15 % of the value measured, actually do not influence the final result (see for example Glagolevskij et al., 1988, 1989). The allowance for some errors requires much observational time to determine the sizes of a star image, to measure the modulation degree, signal-to-noise (S/N) ratio, and magnetic sensitivity coefficient. The account of S/N ratio is necessary when working with objects fainter than m=8-9: white dwarfs, polars, galaxies, the observation of which has been started recently. Expenditures of the observational time on measuring modulation degree and seeing can be fully justified by the higher requirement of measurement accuracy for some observational problems, eg. accurate measurement of periods, search for long-term variability, etc.

MODULATION DEGREE AND S/N RATIO

Modulation degree M is measured from observations of stars on the hydrogen magnetometer of the 6 m telescope (Shtol', 1991) in the single-beam mode. In order to measure it, before the polarimetric analyzer we install a polarizer, transforming the natural star light into the 100 % left-circularly polarized one. As the number of impulses N^+ and N^- proportional to the left- and right-circularly polarized light fluxes is registered in the channels of the magnetometer, and the modulation degree is defined by the expression:

$$M = 100\cdot(N^+-N^-)/(N^++N^-)\ [\%],$$

then at perfect operation of the magnetometer and with the absence of noise M=100 %. As a rule, these conditions are not fulfilled, even at a sufficiently low background for one hydrogen line M=92 %. For two lines (H_β and H_γ) the modulation is less and depends on the energy distribution in the stellar spectrum and on the character of spectral transmission of the magnetometer. Decrease of modulation degree depends on the wavelength and control voltage value supplied to the electrooptical phase element of the polarimetric analyzer.

The modulation coefficient carries one of the systematic errors of magnetic field (polarization) measurement and could be found as K_M = 100/M. The observational result and the error of its measurement is multiplied by this coefficient.

If the modulation degree is measured from a faint star, then the value of the modulation will be underestimated. The true value is defined taking into account a background contribution:

$$M = M_b\cdot K_b,$$

where M_b is the measured modulation degree, $K_b=1/(1-N_b/N_s)$ - the background (noise) coefficient allowing for the decrease of the modulation degree (Shtol', 1991), N_b - the background intensity, N_s the intensity of the light flux from the star [impulses / sec].

At measurements of the magnetic field (polarization) the polarizer is removed from the light path of the magnetometer, and S/N ratio may change (eg. due to decrease of influence on the background of dark impulses of the photomultiplier). Therefore, just before observation of a faint star, it is necessary to determine the noise coefficient K_b and then multiply by it the obtained observational result and the error of its measurement.

STAR IMAGE SIZE

The size α (in arcsec) of a star image (FWHM) at the entrance slit of the hydrogen magnetometer is determined from the following algorithm. The intensities of light fluxes N_1 and N_2 [impulses/sec] are measured at two widths S_1 and S_2 [mm] of the entrance slit ($S_1 > S_2$). The ratio of these intensities $A = N_1/N_2$ is found. For the widths S_1 and S_2 the ratio of functions $B_i = \Phi(S_1, \alpha_i)/\Phi(S_2, \alpha_i)$ is calculated and the value, which is closer to the ratio A is selected from the series B_i. The size of the star image $\alpha_1 \approx \alpha$ corresponds to the selected B_i.

The function $\Phi(S,\alpha) = \Phi(x)$ is defined from the known approximation (eg. Korn and Korn, 1978):

$$\Phi(x) = 1-1/((((a_4 x + a_3)x + a_2)x + a_1)x + 1)^4, \tag{1}$$

where a_1=0.278393, a_2=0.230389, a_3=0.000972, a_4=0.078108, $x_1 = x = (0.833 \cdot S \cdot \beta)/\alpha$ (Shtol', 1991), where β (arcsec per mm) is the image scale at the entrance slit.

In order to avoid an additional systematic error it is desirable to measure N_1 and N_2 in the region of the continuum, since the instrumental contour of a spectral line depends on the width of the entrance slit.

LOSSES AT THE ENTRANCE SLIT OF THE MAGNETOMETER

The losses are determined from a given or known seeing value α for a given slit, its width S [mm] and height H [mm] (the height of the entrance slit for one image, corresponding to the exit slit projection on the entrance slit allowing for the spectrograph (broad slit operation) used in the magnetometer as a monochromator).

The coefficient of the losses is defined by the expression:

$$K_{SN} = \Phi(x_1) \cdot \Phi(x_2), \tag{2}$$

or by

$$K_{SN} = N_S/N_0,$$

where N_S is the light flux intensity at the given slit size, N_0 is the intensity of the light flux [impulses/sec], falling on the entrance slit, $x_2 = (0.833 \cdot H \cdot \beta)/\alpha$; and $\Phi(x_1)$ and $\Phi(x_2)$ are calculated from Eq (1).

The coefficient, allowing for the increase of the magnetic field (polarization) measurement error due to the losses at the magnetometer entrance slit

$$K_S = K_{SN}^{-1/2}, \tag{3}$$

may be used for object selection for observations, as well as for determination of the optimal slit size.

ERROR CAUSED BY SEEING VALUE

This error arises in the optical path of the magnetometer as a result of some mixing of two beams at the entrance slit.

Fig. 1 shows the projection of the magnetometer exit slits in the focal plane of the polarimetric analyzer, where two star images are constructed after their light passed through the polarization splitter of the analyzer.

Introducing the indications:

$\Phi(x_3)$, $\Phi(x_4)$, $x_3 = (0.833\cdot\beta\cdot(2C + H))/\alpha$, $x_4 = (0.833\cdot\beta\cdot(2C - H))/\alpha$,

and the error coefficient K_α depending on the seeing value, as in the expression for the modulation degree one can obtain from Fig. 1:

$K_\alpha = (1+K)/(1-K)$,

where $K = 0.5\cdot[\Phi(x_3)-\Phi(x_4)]/\Phi(x_2)$.

The functions $\Phi(x_3)$and $\Phi(x_4)$ are calculated from Eq (1).

In order to take into account the error caused by the seeing, one should multiply the result (magnetic field - polarization) and its measurement error obtained at the image α by the coefficient K_α.

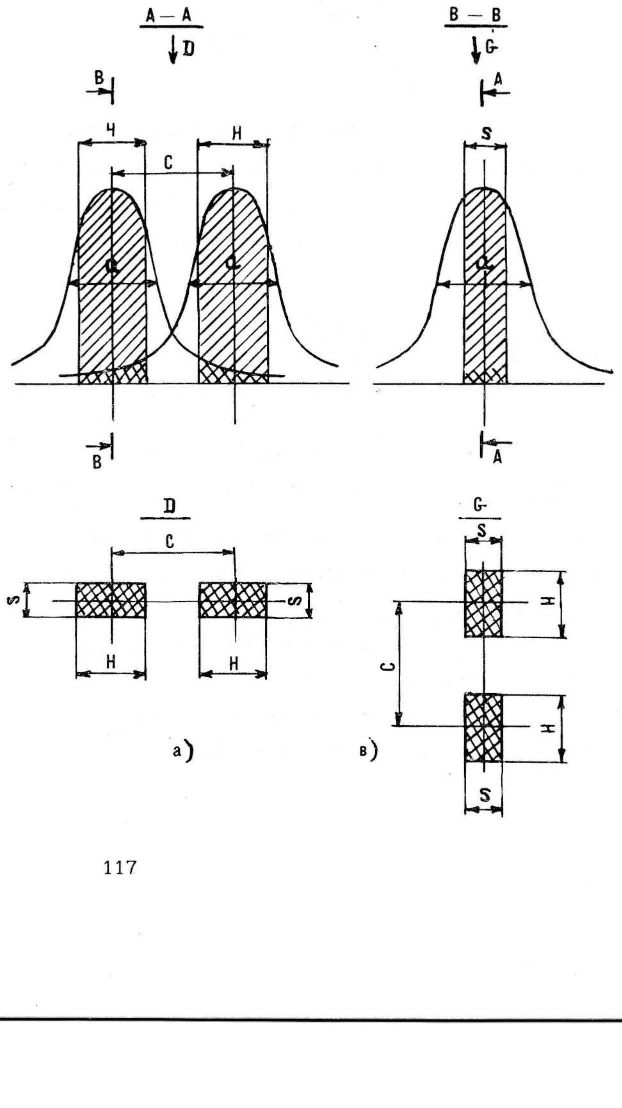

Fig. 1. *The projection of the magnetometer exit slits onto the focal plane of the polarimetric analyzer and on the brightness distribution of star images constructed in this plane.*
a) section A-A and top view D;
b) section B-B 'and top view G;
α is a seeing [FWHM],
S - width, H - height,
C - distance between the slits centres.

The same procedure should be done when measuring the modulation degree and calculating its coefficient:

$M = K_\alpha \cdot M_\alpha$ and $K_M = 100/K_\alpha/M_\alpha$,

where M_α is the modulation degree (depth), measured at the image size α, and M and K_M are actual values of modulation and its coefficient.

MAGNETIC SENSITIVITY COEFFICIENT

Magnetic sensitivity coefficient is used to calculate the values and errors of the measured magnetic field longitudinal component. The magnetic field is a signal $(N^+ - N^-)/(N^+ + N^-)$, multiplied by this coefficient (Shtol', 1991), where N^+, N^- - informational and sample channel fluxes.

The derivation of approximate formulae for magnetic sensitivity coefficient K_B was reported by Shtol' (1991).

If a part of the line profile, restricted by the slit (Fig.2), is close to a straight line, then K_B can be calculated by:

$$K_B = 1.072 \cdot 10^{12} \frac{\Delta\lambda_s (I_d + I_e)}{\lambda_0^2 \cdot g \cdot (I_d - I_e)} \text{ [Gs]} \tag{4}$$

where $\Delta\lambda_s$ is the slit width [Å], λ_0- central line wavelength [Å], g - Lande factor, I_d- intensity at the outer slit edge, I_e- intensity at the inner slit edge.

For the real line profile it is possible also to find K_B for the "blue" and "red" wings of, for example, two hydrogen lines H_β and H_γ (g = 1) (see Fig. 2):

$$K_B = 1.072 \cdot 10^{12} \cdot \frac{\Delta\lambda_S}{n} \cdot \frac{I_1 + I_2 + I_3 + I_4 + 2 \cdot \sum_{i=2}^{n} I_i}{\lambda_{0\beta}^2 (I_1 - I_2) + \lambda_{0\gamma}^2 (I_3 - I_4)} \text{ [Gs]} \tag{5}$$

where $I_1 = I_{\beta B1} + I_{\beta R1}$; $I_2 = I_{\beta B(n+1)} + I_{\beta R(n+1)}$; $I_3 = I_{\gamma B1} + I_{\gamma R1}$; $I_4 = I_{\gamma B(n+1)} + I_{\gamma R(n+1)}$; $I_i = I_{\beta Bi} + I_{\beta Ri} + I_{\gamma Bi} + I_{\gamma Ri}$, where i=1,2,3,...,n; n is the number of equal spectral intervals forming the part of line profile restricted by the slit; $I_{\beta B1}$- flux intensity on the outer slit edge, positioned in the "blue" wing of the line H_β; $I_{\beta R1}$- the same for the "red" wing; $I_{\beta B(n+1)}$, $I_{\beta R(n+1)}$- the same for the inner slit edge; $I_{\gamma B1}$, $I_{\gamma R1}$, $I_{\gamma B(n+1)}$, $I_{\gamma R(n+1)}$- the same for the line H_γ; $I_{\beta Ri}$, $I_{\beta Bi}$, $I_{\gamma Ri}$- intermediate intensity values (in division points).

As a particular case of (5) is the expression for the magnetic sensitivity coefficient for one wing of one hydrogen line:

$$K_B = 1.072\cdot10^{12}\cdot\frac{\Delta\lambda_s}{\lambda_0^2\cdot n}\cdot\frac{I_1+I_{n+1}+2\cdot\sum_{i=2}^{n} I_i}{I_1-I_{n+1}} \quad [\mathrm{Gs}] \tag{6}$$

This formula is more accurate than that presented by Shtol' (1991). We would like to point out the mistake appeared in Eq (6) (Shtol', 1991): the number of spectral intervals in denominator must be squared (n^2) there.

The K_B can be calculated with a sufficient accuracy from a profile, obtained by photoelectric scanning by the magnetometer entrance slit.

To find K_B is also possible from the spectrogram records with a preliminary folding of the line profiles with the instrumental magnetometer contour. The accuracy of such calculations depends on spectral data quality and its correction for the magnetometer spectral characteristic (if such a correction is needed).

Combination of the methods mentioned above allows to minimize the observing time for determination of K_B by using the formula derived from (4) and (6):

$$K_B=1.072\cdot10^{12}\cdot\frac{\Delta\lambda_s}{\lambda_0^2}\cdot\frac{I_1+I_{n+1}}{I_1-I_{n+1}}\cdot K_\Delta \quad [\mathrm{Gs}] \tag{7}$$

where $K_\Delta=(1+2\cdot\sum_{i=2}^{n} I_i/(I_1+I_{n+1}))/n$ is the coefficient allowing for nonlinearity of the line profile part, restricted by the slits, and defined from the spectrogram, I_1 and I_{n+1} - intensities at the external and internal slit edges, measured by positioning the magnetometer slit in the appropriate place of a line wing.

THE OPTIMAL SLIT SIZE

The magnetometer optimal slit size and its location on the line wing are defined from spectrograms. For this purpose the line wing is divided into equal spectral intervals. The size of the slit $\Delta\lambda_s$ is selected. The intensity values (in division points) for the profile, corrected for the slit size are calculated. The profile at a given slit size can be defined by the numerical integration method, eg. trapezium method:

$$I_s = (I_1+I_{n+1}+2\cdot\sum_{i=2}^{n} I_i)/2n,$$

where I_s is the intensity value after the integration within the slit limits, n - the number of intervals, falling within the slit. Then, moving the slit along the line wing and changing its size the series of values of σ_B - some errors of the magnetic field measurement - is calculated by Eq (8) derived from (6) of (Shtol', 1991). These errors are determined by magnetic sensitivity coefficients and by relative light fluxes in the slit (relative line profile intensities in the points of its division):

$$\sigma_B = \frac{\sqrt{\Delta\lambda_s \cdot (I_1 + I_{n+1} + 2\cdot\sum_{i=2}^{n} I_i)}}{\sqrt{n}\,(I_1 - I_{n+1})} \qquad (8)$$

The optimal slit position in the line wing and its optimal size correspond to the minimum value of σ_B from the whole series of values, defined from Eq (8). The distance between the centres of the line and the slit is determined by the size of the latter, by the accuracy of its positioning with respect to the line centre and by the possibility to compensate for its displacement due to the telescope movement in z direction. When selecting a minimum slit size it is necessary to take into account observational conditions - possible losses of light at the magnetometer entrance slit. For this purpose the star size is specified (for the 6 m telescope statistically average seeing is $\alpha\approx3''$) and the values of the whole series of σ_B, defined from Eq (8) are multiplied by K_s coefficients, found from Eqs (2) and (3) for all slit sizes ($\Delta\lambda_s \equiv S$) used in the determination of the σ_B series.

ERROR CAUSED BY SHIFT OF THE SLIT AS A FUNCTION OF FLEXURE AND ZENITH ANGLE

As a result of flexure the entrance slits of the magnetometer will displace (shift) along the spectra of both beams constructed in the focal plane of the spectrograph in the process of zenith motion of the telescope: i.e. $\Delta\lambda_z = \Delta\ell_z \cdot d\lambda/d\ell = f(z)\cdot d\lambda/d\ell$, where $\Delta\lambda_z$ is the displacement of the slits in Angstroms, $\Delta\ell_z$ is the linear displacement of the slits in mm, $d\lambda/d\ell$ is the reciprocal linear dispersion of the spectrograph as a part of the magnetometer, z is the zenith angle.

When changing z (according to Fig. 2), the slits are shifted with respect to the line centre - one of them is approaching the centre and the other is moving away from it. It effects the magnetic field measurement accuracy, since the magnetic sensitivity coefficient and the intensity of the light flux, passing through the slits, change. But if the slit sizes, the distance between them and manufacturing tolerances are not properly selected, then the shift of the slits may cause suppression of the valid signal - one of the slits moves beyond the line centre. Therefore for the selected size and position of the slit there should be found a critical shift $\Delta\lambda_{max}$ [Å], or the same expressed in linear measure:

$$\Delta\ell_{max} = (\ell_{RB} - \ell_s)/2 - \Delta\ell_{RB} - \Delta\ell_s \text{ [mm]},$$

where ℓ_{RB} is the distance between the centres of the slits, positioned in the spectra of the first and second beams in the "red" and "blue" line wings [mm] respectively, ℓ_s - the slit size [mm], $\Delta\ell_{RB}$ and $\Delta\ell_s$ - margin slit size tolerance [mm].

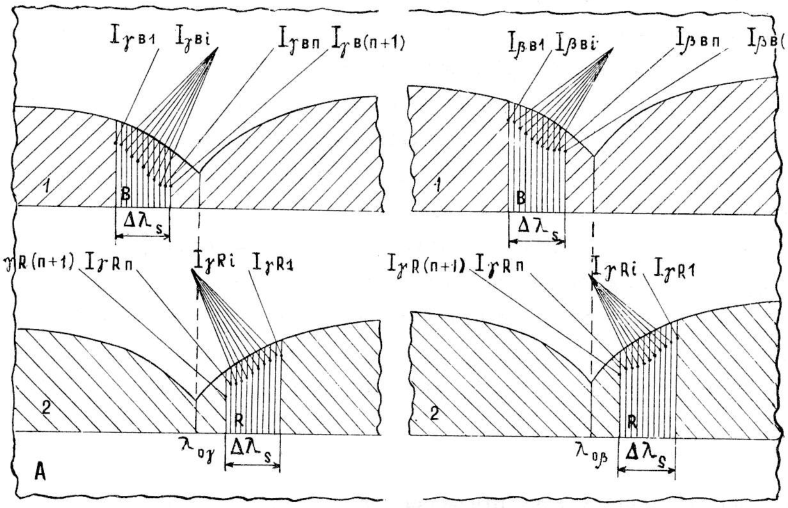

Fig. 2. *Position of the magnetometer exit slits relatively to the wings of H_γ and H_β lines. A is the focal plane of the spectrograph chamber of the hydrogen magnetometer; 1 - graphic presentation of the spectral image of the first magnetometer beam; 2 - the same for the second beam; $\lambda_{0\gamma}$ - centre of the line H_γ; $\lambda_{0\beta}$ - the same for H_β; B, R - position of the magnetometer exit slits for "blue" and "red" wings of the lines H_γ and H_β, respectively; $\Delta\lambda_s$ - the width of the entrance slits; $I_{\gamma B1}$ - the intensity of light flux at the external slit edge, placed in the "blue" wing of the line H_γ; $I_{\gamma R1}$ - the same for the "red" wing; $I_{\gamma B(n+1)}$, $I_{\gamma R(n+1)}$ - the same for the internal slit edge; $I_{\beta B1}$, $I_{\beta R1}$, $I_{\beta B(n+1)}$, $I_{\beta R(n+1)}$ - the same for the line H_β; $I_{\gamma Bi}$, $I_{\gamma Bn}$, $I_{\gamma Ri}$, $I_{\gamma Rn}$, $I_{\beta Bi}$, $I_{\beta Bn}$, $I_{\beta Ri}$, $I_{\beta Rn}$ - intermediate values of the intensities.*

The degree of displacement of slits $\Delta\ell_z$ and $\Delta\lambda_z$ as a function of z can be estimated by experiment for various configurations of magnetometer light detectors. For the configuration, which is more frequently used, the values of $\Delta\ell_z$ are presented in Table 1. The zenith angle $z = 60^\circ$, at which the slits are positioned symmetrically about the line centre (Shtol', 1984, 1991) corresponds to the zero shift of the slits.

Table 1. Displacement of the magnetometer slits as a result of telescope motion.

z°	0	10	20	30	40	45	50	55	60	65	70	75	80
$\Delta\ell_z$ mm	-0.35	-0.33	-0.30	-0.25	-0.19	-0.15	-0.10	-0.05	0.00	+0.07	+0.15	+0.25	+0.37

The magnetometer has a mechanical device, which facilitates corrections of flexure by shifting the slits to a set distance. The scale multiplier of the linear shift indicator is 0.01 mm, and the size of the slits (close to minimum for the existing scheme of the magnetometer) is 0.3 mm (4.5 Å). Further reduction of the slits causes considerable light losses or requires very good observing conditions. For example, for the slits of 0.2 mm (3 Å) the seeing must not exceed 1″.

The distance between the slits is selected on the basis of the optimal slit condition and the possibility to correct slit shifts with variation of z. The correction (by the mechanical device) is made in the process of observations with an accuracy, defined by the measurement error $\Delta\ell_z = f_1(z)$.

Possible shift without correction is conditioned by the required magnetic field measurement accuracy, depending on an observational problem (eg. for detection of magnetic field the precision requirements are lower than for measurement of its intensity).

Admissible accuracy variation might be given in this form:

$$K_{\sigma z}=\sigma_{\Delta z}/\sigma_z$$

where $K_{\sigma z}$ is the coefficient of possible accuracy variation, σ_z is the relative accuracy of magnetic field measurement, defined from Eq (8), transformed for calculations from two line wings (see Eq (5)) with unshifted slits, $\sigma_{\Delta z}$ is the same with shifted slits.

The spectrograms from which the optimal slit size has been determined can be used for the σ_z and $\sigma_{\Delta z}$ calculations.

The possible value of the slit shift $\Delta\ell_{\sigma z}$ is defined by the same methods the optimal slit is. But the calculation is made for both wings simultaneously. $\Delta\ell_{\sigma z}$ is defined by minimum difference of the given value of $K_{\sigma z}$ from the calculated one, which is selected from the set of $\sigma_{\Delta z}/\sigma_z$ ratios, with $\sigma_{\Delta z}$ being obtained at discrete slit shift from the position at which σ_z is found. A step of the discrete slit shift is selected to be equal to that of the line division by intervals. In much the same way one can define tolerances for the slit sizes and their location on the mask (changeable device carrying exit slits of the magnetometer).

While measuring the magnetic field the error may be connected with the relation of seeing and the magnetometer entrance slit width.

If the image size is considerably smaller than the entrance slit, then the line profile (and therefore - the magnetic sensitivity coefficient) is determined exactly by the seeing. In this case the accuracy of the magnetic field measurement depends on the star image and its motion in the slit. The simplest way how one can get rid of the error is to defocus the telescope. Hereat the conditions should be followed: defocussing must not increase essentially the losses at the slit and be made due to the telescope focal length extension (in this case the modulation depth remains actually unchanged).

CONCLUSIONS

Allowance for the mentioned errors at the existing scheme of the hydrogen magnetometer requires additional observational time for their determination, but for the error caused by flexure of the magnetometer light detectors and depending only on the accuracy and reproducibility of determination of flexure as a function of zenith angle of the telescope. Some modifications in the optical path and in the scheme of the magnetometer will allow to reduce considerably (actually eliminate) the error associated with the seeing, and to measure modulation degree in the process of pointing on an object, or moving the telescope from object to object. In the first case it is the placing of the analyzer or even its polarization splitter after the entrance slit, in the second case it is the setting of an artificial star in the path of the magnetometer.

The optimal way of determination and calculation of the magnetic sensitivity coefficient from the point of view of observational time expenditure (if the calculation from spectrograms does not provide sufficient accuracy and reliability) is the version based on the application of Eq (7), allowing to minimize the number of measures i.e. two for each line wing.

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